

Summer Work Packet For Students Entering Algebra 1 Honors

June 2023

Dear Student,

Welcome! I have prepared a summer work packet for you to help you better prepare for your upcoming course, Algebra 1 Honors. As academic standards become more rigorous as a result of the implementation of the Common Core State Standards, I would like my students to be able to demonstrate and communicate an in-depth understanding of the topics taught in mathematics. My goal is not only to have the students master a particular skill, but also to be able to apply these skills in real-life situations.

As you prepare to take Algebra 1 Honors, this booklet covers *some* of the many skills that you should have been exposed to during your previous years of schooling. These skills continue to be used in the study of Algebra 1 and the more confident and proficient you are with them, the easier it will be to learn new concepts.

This is NOT a test! If you encounter some material that is not familiar to you, or you do not remember how to do a particular problem(s) – don't panic. You may use any resource (i.e. a family member, friend, the internet) to help you.

When working through these problems, your focus should be on not only getting the correct answer, but also developing good habits. The following are some things that should sound familiar to you:

- Show and label all your work – even when a calculator is used
- Keep your work neat and organized
- Don't cram your work on the paper if there is not enough room – use another piece of paper!
- Neatly erase any corrections (pencil is preferred)
- Write complete sentences when answering word problems
- **It is ok to be wrong** - just be able to explain where you went wrong, and don't erase what you have done.

We look forward to a great school year with you. Enjoy your summer!

Sincerely,
Ms. Nhotsoubanh

Note* This work is optional.**

This course requires the use of the TI-84 Plus graphing calculator. It will be provided to you but I highly recommend purchasing one of your own. You will continue to use the graphing calculator throughout your high school and college years. Check out the weekly ads from Staples for deals. You will need four 1-subject spiral notebooks for this course.



Rename Fractions, Percents, and Decimals

Hints/Guide:

To convert fractions into decimals, we start with a fraction, such as $\frac{3}{5}$, and divide the numerator (the top number of the fraction) by the denominator (the bottom number of the fraction). So:

$$5 \overline{)3.0} \quad \text{and the fraction } \frac{3}{5} \text{ is equivalent to the decimal } 0.6$$

To convert a decimal to a percent, we multiply the decimal by 100 (percent means a ratio of a number compared to 100). A short-cut is sometimes used of moving the decimal point two places to the right (which is equivalent to multiplying a number by 100), so $0.6 \cdot 100 = 60$ and $\frac{3}{5} = 0.6 = 60\%$.

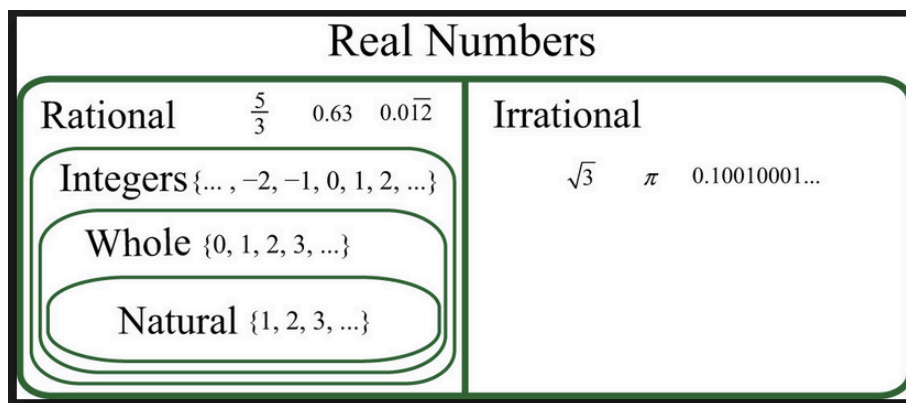
To convert a percent to a decimal, we divide the percent by 100,
60% is the same as $60 \div 100$, which is 0.6, so $60\% = 0.6$

To convert a fraction into a percent, we can use proportions to solve, so

$$\frac{3}{5} = \frac{x}{100} \text{ and using cross products to solve, } 5x = 300 \text{ or } x = 60\%$$

Exercises: Complete the chart

	Fraction	Decimal	Percent
1.		0.04	
2.			125%
3.	$\frac{2}{3}$		
4.		1.7	
5.			0.6%
6.	$3\frac{1}{2}$		
7.		0.9	
8.			70%
9.	$\frac{17}{25}$		
10.		0.007	



The **Real** number system is made up of two main sub-groups **Rational numbers** and **Irrational numbers**.

The set of rational numbers includes several subsets: **natural numbers, whole numbers, and integers**.

- **Real Numbers**- any number that can be represented on a number-line.
 - **Rational Numbers**- a number that can be written as the ratio of two integers (this includes decimals that have a definite end or repeating pattern)

Examples: 2, -5, $\frac{-3}{2}$, $\frac{1}{3}$, 0.253, $0.\overline{3}$

 - **Integers**- positive and negative whole numbers and 0
Examples: -5, -3, 0, 8 ...
 - **Whole Numbers** – the counting numbers from 0 to infinity
Examples: { 0, 1, 2, 3, 4,}
 - **Natural Numbers**- the counting numbers from 1 to infinity
Examples: { 1, 2, 3, 4... }
 - **Irrational Numbers**- Non-terminating, non-repeating decimals (including π , and the square root of any number that is not a perfect square.)
Examples: 2π , $\sqrt{3}$, $\sqrt{23}$, 3.21211211121111....

Practice: Name all the sets to which each number belongs.

1. -4.2 _____

4. 9 _____

2. $3\sqrt{5}$ _____

5. $\sqrt{16}$ _____

3. $\frac{5}{3}$ _____

6. $-\frac{8}{2}$ _____

Laws of Exponents

Hints/Guide:

There are certain rules when dealing with exponents that we can use to simplify problems. They are:

Adding powers $a^m a^n = a^{m+n}$

Multiplying powers $(a^m)^n = a^{mn}$

Subtracting powers $\frac{a^m}{a^n} = a^{m-n}$

Negative powers $a^{-n} = \frac{1}{a^n}$

To the zero power $a^0 = 1$

Here are some examples of problems simplified using the above powers:

$$4^3 \bullet 5^5 = 4^8 \quad (4^3)^3 = 4^9 \quad 4^5 \div 4^3 = 4^2 \quad 4^{-4} = \frac{1}{4^4} = \frac{1}{256} \quad 4^0 = 1$$

Exercises: Simplify the following problems using exponents (Do not multiply out).

1. $5^2 5^4 =$

2. $7^{-3} 7^5 =$

3. $(12^4)^3 =$

4. $(6^5)^2 =$

5. $5^9 \div 5^4 =$

6. $10^3 \div 10^{-5} =$

7. $7^{-3} =$

8. $3^{-4} =$

9. $124^0 =$

10. $-9^0 =$

11. $(3^5 \bullet 3^2)^3 =$

12. $5^3 \bullet 5^4 \div 5^7 =$

Find Percent of a Number

Hints/Guide:

To determine the percent of a number, we must first convert the percent into a decimal by dividing by 100 (which can be short-cut by moving the decimal point in the percentage two places to the left), then multiplying the decimal by the number.

$$\text{Percent Equation} \Rightarrow \text{is} = \% \text{ (of)} \quad \text{or} \quad \text{Percent Proportion} \Rightarrow \frac{\%}{100} = \frac{\text{is}}{\text{of}}$$

For example:

$$\begin{aligned} 4.5\% \text{ of } 240 \text{ start with formula: } \text{is} &= \% \text{ (of)} \\ &= 4.5\% \cdot 240 \\ &= 0.045 \cdot 240 \\ &= 10.8 \end{aligned}$$

Answer: 4.5% of 240 is 10.8.

$$\begin{aligned} \frac{\%}{100} &= \frac{\text{is}}{\text{of}} \\ \frac{4.5}{100} &= \frac{x}{240} \\ 100x &= 4.5(240) \\ \frac{100x}{100} &= \frac{1080}{100} \\ x &= 10.8 \end{aligned}$$

SHOW ALL WORK.

1. 7.5% of 42 is what number?	2. 18 is what percent of 120?
3. 12% of what number is 54?	4. 8% of 20 is what number?
5. 96 is what percent of 80?	6. 3.75 is 5% of what number?

Solving Equations

Hints/Guide:

As we know, the key in equation solving is to isolate the variable. In equations with variables on each side of the equation, we must combine the variables first by adding or subtracting the amount of one variable on each side of the equation to have a variable term on one side of the equation. Then, we must undo the addition and subtraction, then multiplication and division.

To solve an equation with the same variable on each side, write an equivalent equation that has the variable on just one side of the equation. Then solve.

Example Solve $4(2a - 1) = -10(a - 5)$.

$$4(2a - 1) = -10(a - 5)$$

Original equation

$$8a - 4 = -10a + 50$$

Distributive Property

$$8a - 4 + 10a = -10a + 50 + 10a$$

Add $10a$ to each side.

$$18a - 4 = 50$$

Simplify.

$$18a - 4 + 4 = 50 + 4$$

Add 4 to each side.

$$18a = 54$$

Simplify.

$$\frac{18a}{18} = \frac{54}{18}$$

Divide each side by 18.

$$a = 3$$

Simplify.

The solution is 3.

Practice: Solve each equation.

1. $5 + 3r = 5r - 19$

2. $8x + 12 = 4(3 + 2x)$

3. $-5x - 10 = 2 - (x + 4)$

4. $6(-3m + 1) = 5(-2m - 2)$

5. $3(d - 8) - 5 = 9(d + 2) + 1$

Summer Mathematics Packet

Algebraic Translations

Hints/Guide:

Key Words for Translations:

Add	Subtract	Multiply	Divide	Inequalities	Variable	=
Plus Sum Longer Than Greater Than Together Total Increased More Than In all And	Decreased Smaller Less than Difference Reduced Differ Fewer Shorter Than Minus Diminished	Per For Every For each Triple Multiplied Of Times Twice Double	One-third Quotient Divided by Each part Half as much Spilt equally	$<$ is less than $>$ is greater than \leq is less than or equal to \geq is greater than or equal to	a number some number quantity	Same as Equals Is Total Was Result Outcome Answer

Practice: Translate each phrase into a mathematical statement

1. Seven plus five times a number is greater than or equal to -9
2. Eight times a number increased by 6 is 62
3. One half of a number is equal to 14
4. 6 less than 8 times some number
5. a number divided by 9
6. p decreased by 5
7. twice a number decreased by 15 is equal to -27
8. 9 less than 7 times some number is -6
9. the sum of a number and eight is less than 2
10. eleven increased by a number is -12

Word Problems

Hints/Guide:

Translate each word problem into an algebraic equation, using x for the unknown, and solve. Write a “**let $x =$** ” for each unknown; write an equation; solve the equation; substitute the value for x into the let statements(s) to answer the question.

For Example:

Kara is going to Maui on vacation. She paid \$325 for her plane ticket and is spending \$125 each night for the hotel. How many nights can she stay in Maui if she has \$1200?

Step 1: What are you asked to find? Let variables represent what you are asked to find.

How many nights can Kara stay in Maui?

Let $x =$ The number of nights Kara can stay in Maui

Step 2: Write an equation to represent the relationship in the problem.

$$325 + 125x = 1200$$

Step 3: Solve the equation for the unknown

$$\begin{array}{r} 325 + 125x = 1200 \\ - 325 \quad \quad -325 \\ \hline 125x = 875 \\ x = 7 \end{array}$$

Kara can spend 7 nights in Maui

Word Problem Practice Set

1. A video store charges a one-time membership fee of \$12.00 plus \$1.50 per video rental. How many videos can Stewart rent if he spends \$21?
2. Bicycle city makes custom bicycles. They charge \$160 plus \$80 for each day that it takes to build the bicycle. If you have \$480 to spend on your new bicycle, how many days can it take Bicycle City to build the bike?
3. Darel went to the mall and spent \$41. He bought several t-shirts that each cost \$12 and he bought 1 pair of socks for \$5. How many t-shirts did Darel buy?

Volume/ Surface Area

Hints/Guide:

The formulas we need to know are:

Volume of a rectangular prism: $V = lwh$

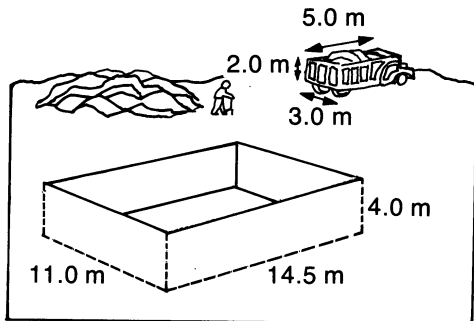
Volume of a cylinder: $V = \pi r^2 h$

Surface Area of a rectangular prism: $SA = 2lw + 2wh + 2hl$

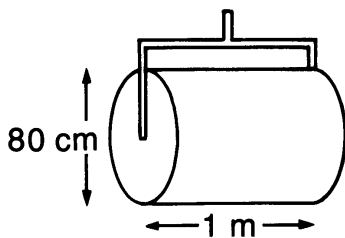
Surface Area of a cylinder: $SA = 2\pi rh + 2\pi r^2$

Exercises: Write out the formula and show the substitutions.

1. The excavation for a house and the trucks to carry away the material, have the dimensions shown. About how many level truck loads are necessary to remove all the dirt?



2. A lawn roller is 1 m wide and 80 cm high. What area is covered in each revolution?



Coordinate Geometry

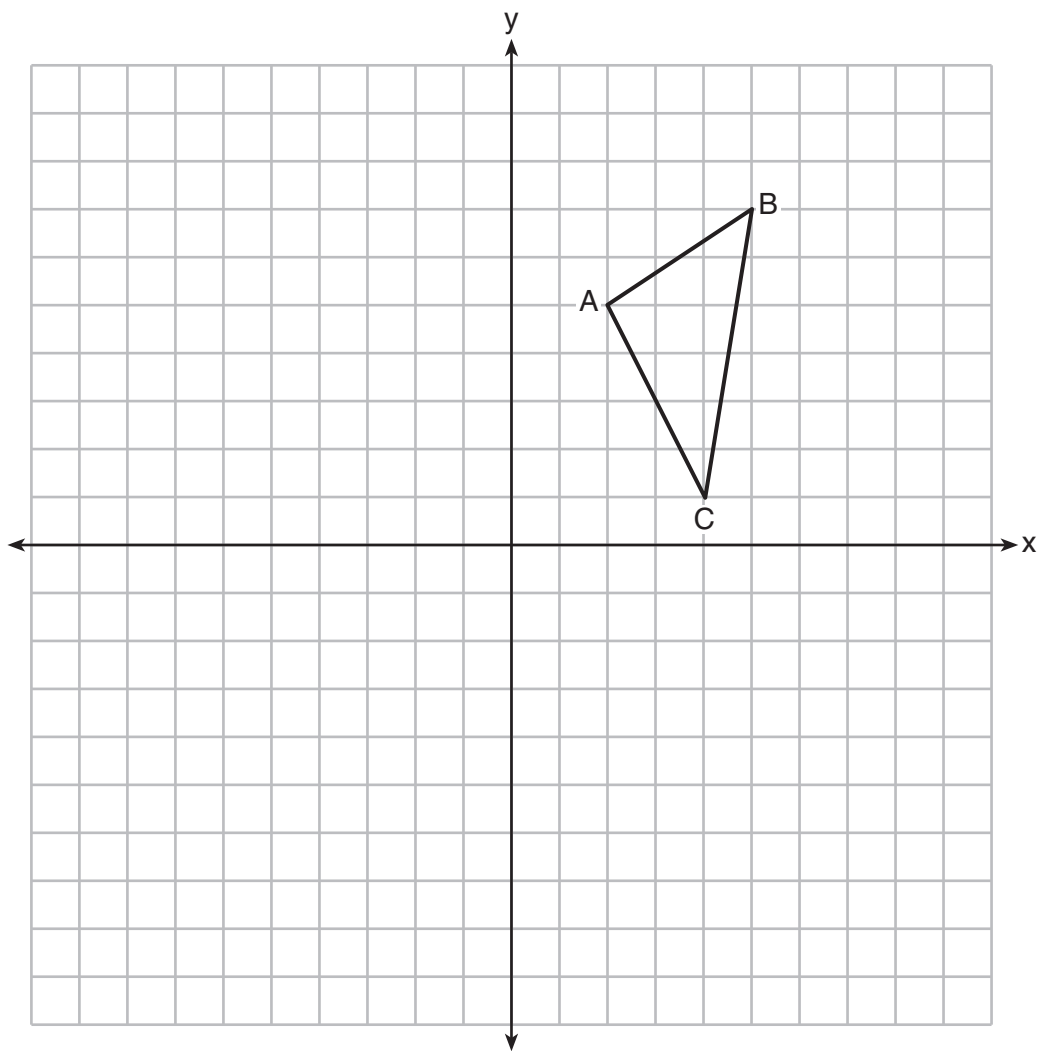
Use a ruler.

The coordinates of $\triangle ABC$, shown on the graph below, are $A(2, 5)$, $B(5, 7)$, and $C(4, 1)$.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after it is reflected over the y -axis.

Graph and label $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after it is reflected over the x -axis.

State a single transformation that will map $\triangle ABC$ onto $\triangle A''B''C''$.



Graphing Linear Equations

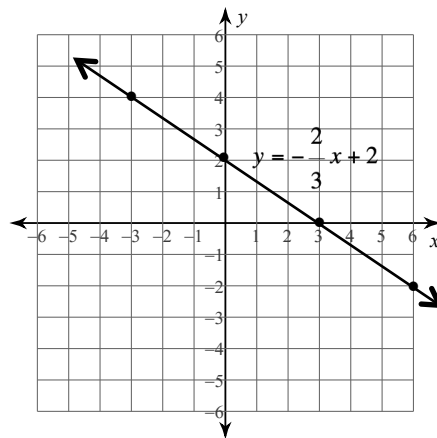
Hints/Guide:

Graphing a linear equation in slope-intercept form: $y = mx + b$,
where m = slope and b = y-intercept.

Example: $y = -\frac{2}{3}x + 2$

$$m = -\frac{2}{3}$$

$$b = 2$$

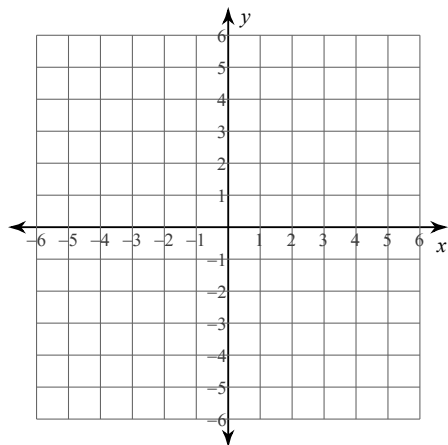


Directions: Graph the given line. Use a straightedge to connect the points. You should have at least 3 points.

1. $y = x + 2$

slope(m) = _____

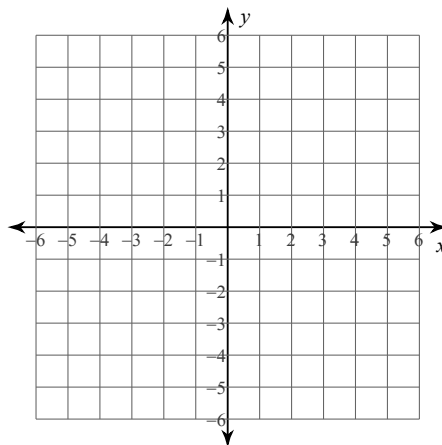
y-intercept(b) = _____



2. $y = 2x - 3$

slope(m) = _____

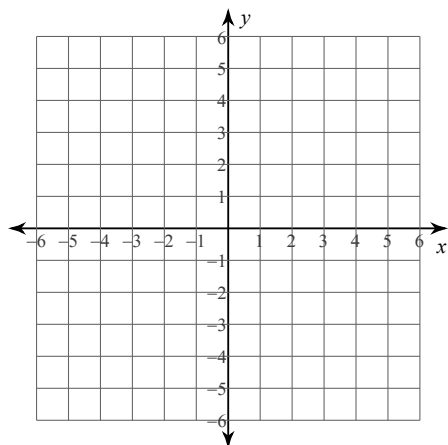
y-intercept(b) = _____



3. $y = \frac{1}{2}x + 1$

slope(m) = _____

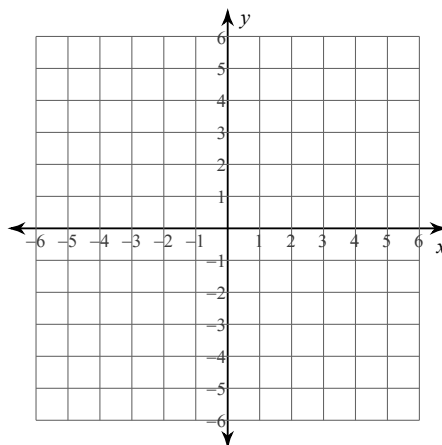
y-intercept(b) = _____



4. $y = \frac{3}{2}x - 1$

slope(m) = _____

y-intercept(b) = _____



Inequalities

Hints/Guide:

An inequality is a statement containing one of the following symbols:

$<$ is less than

$>$ is greater than

\leq is less than or equal to

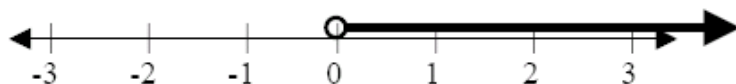
\geq is greater than or equal to

An inequality has many solutions, and we can represent the solutions of an inequality by a set of numbers on a number line.

When graphing an inequality, $<$ and $>$ use an open circle \circ \leq and \geq use a closed circle \bullet

Examples:

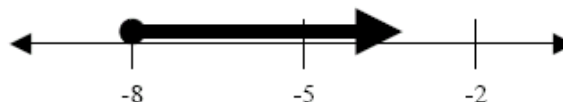
$$x > 0$$



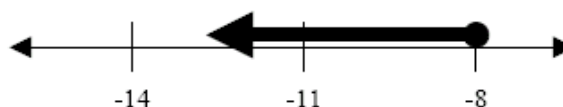
$$x < 0$$



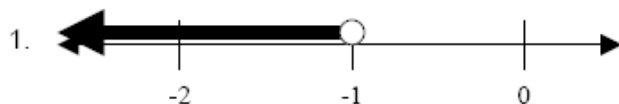
$$x \geq -8$$



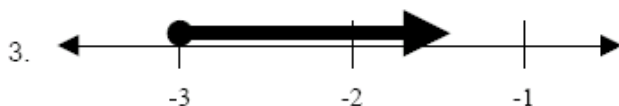
$$x \leq -8$$



Practice: Write an inequality to represent the solution set that is shown in the graph.



2. $5 > y$



4. $-2 \leq t$



Combining Like terms

Terms in algebra are numbers, variables or the product of numbers and variables. In algebraic expressions terms are separated by addition (+) or subtraction (-) symbols. Terms can be combined using addition and subtraction if they are **like-terms**.

Like-terms have the same variables to the same power.

Example of like-terms: $5x^2$ and $-6x^2$

Example of terms that are **NOT** like-terms: $9x^2$ and $15x$

Although both terms have the variable x , they are not being raised to the same power

To combine like-terms using addition and subtraction, add or subtract the numerical factor

Example: Simplify the expression by combining like-terms

$$\begin{aligned} 8x^2 + 9x - 12x + 7x^2 &= (8+7)x^2 + (9-12)x \\ &= 15x^2 + -3x \\ &= 15x^2 - 3x \end{aligned}$$

Practice: Simplify each expression

1. $5x - 9x + 2$

2. $3q^2 + q - q^2$

3. $c^2 + 4d^2 - 7d^2$

4. $5x^2 + 6x - 12x^2 - 9x + 2$

5. $2(3x - 4y) + 5(x + 3y)$

6. $10xy - 4(xy + 2x^2y)$

Properties of Real Numbers

Following are properties of Real Numbers that are useful in evaluating and solving algebraic expressions.

Additive Identity	For any number a , $a + 0 = a$.
Multiplicative Identity	For any number a , $a \cdot 1 = a$.
Multiplicative Property of 0	For any number a , $a \cdot 0 = 0$.
Multiplicative Inverse Property	For every number $\frac{a}{b}$, $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Reflexive Property	For any number a , $a = a$.
Symmetric Property	For any numbers a and b , if $a = b$, then $b = a$.
Transitive Property	For any numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then a may be replaced by b in any expression.
Commutative Properties	For any numbers a and b , $a + b = b + a$ and $a \cdot b = b \cdot a$.
Associative Properties	For any numbers a , b , and c , $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Practice: Name the property illustrated in each equation.

1. $3 \cdot x = x \cdot 3$ _____

2. $3a + 0 = 3a$ _____

3. $2r + (3r + 4r) = (2r + 3r) + 4r$ _____

4. $5y \cdot \frac{1}{5y} = 1$ _____

5. $9a + (-9a) = 0$ _____

6. $(10b + 12b) + 7b = (12b + 10b) + 7b$ _____

7. $5x + 2 = 5x + 2$ _____

8. If $9 + 4 = 13$ and $13 = 2 + 11$ then $9 + 4 = 2 + 11$ _____

9. If $x = 7$ then $7 = x$ _____

10. $3 \cdot 1 = 3$ _____

The Distributive Property

The Distributive Property states for any number a , b , and c :

1. $a(b + c) = ab + ac$ or $(b + c)a = ba + ca$

2. $a(b - c) = ab - ac$ or $(b - c)a = ba - ca$

Practice: Rewrite each expression using the distributive property.

1. $7(h - 3)$

2. $-3(2x + 5)$

3. $(5x - 9)4$

4. $\frac{1}{2}(14 - 6y)$

5. $3(7x^2 - 3x + 2)$

6. $\frac{1}{4}(16x - 12y + 4z)$

7. $(9 - 2x + 3xy) \cdot -4$

8. $0.3(40a + 10b - 5)$

Ratios and Proportions

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes**. They are the first and last terms of the proportion. 5 and 10 are called **means**. They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of Proportions

For any numbers a , b , c , and d , if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example 1:

$$\frac{x}{5} = \frac{10}{13}$$

$$x \cdot 13 = 5 \cdot 10$$

$$13x = 50$$

$$\frac{13x}{13} = \frac{50}{13}$$

$$x = \frac{50}{13}$$

Example 2:

$$\frac{x+1}{4} = \frac{3}{4}$$

$$4(x+1) = 3 \cdot 4$$

$$\begin{array}{r} 4x + 4 = 12 \\ -4 \quad -4 \end{array}$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4}$$

$$x = 2$$

Practice: Solve each proportion.

1. $\frac{x}{21} = \frac{3}{63}$

4. $\frac{9}{y+1} = \frac{18}{54}$

2. $\frac{-3}{x} = \frac{2}{8}$

5. $\frac{a-8}{12} = \frac{15}{3}$

3. $\frac{0.1}{2} = \frac{0.5}{x}$

6. $\frac{3+y}{4} = \frac{-y}{8}$

Rate of Change and Slope

Find Slope

Slope of a Line	$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on a nonvertical line
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Example 1 Find the slope of the line that passes through $(-3, 5)$ and $(4, -2)$.

Let $(-3, 5) = (x_1, y_1)$ and $(4, -2) = (x_2, y_2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ &= \frac{-2 - 5}{4 - (-3)} && y_2 = -2, y_1 = 5, x_2 = 4, x_1 = -3 \\ &= \frac{-7}{7} && \text{Simplify.} \\ &= -1 \end{aligned}$$

Example 2 Find the value of r so that the line through $(10, r)$ and $(3, 4)$ has a slope of $-\frac{2}{7}$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ -\frac{2}{7} &= \frac{4 - r}{3 - 10} && m = -\frac{2}{7}, y_2 = 4, y_1 = r, x_2 = 3, x_1 = 10 \\ -\frac{2}{7} &= \frac{4 - r}{-7} && \text{Simplify.} \\ -2(-7) &= 7(4 - r) && \text{Cross multiply.} \\ 14 &= 28 - 7r && \text{Distributive Property} \\ -14 &= -7r && \text{Subtract 28 from each side.} \\ 2 &= r && \text{Divide each side by } -7. \end{aligned}$$

Practice:

Find the slope of the line that passes through each pair of points.

1. $(4, 9), (1, 6)$

3. $(4, 3.5), (-4, 3.5)$

2. $(2, 5), (6, 2)$

4. $(1, -2), (-2, -5)$

Determine the value of r so the line that passes through each pair of points has the given slope.

5. $(6, 8), (r, -2), m = 1$

6. $(10, r), (3, 4), m = -\frac{2}{7}$