# **10**

# **WRITING AND SOLVING SYSTEMS OF LINEAR FUNCTIONS**

Architects often add an outdoor stairway to a building as a design feature or as an approach to an entrance above ground level. But stairways are an obstacle to persons with disabilities, and most buildings are now approached by means of ramps in addition to or in place of stairways.

In designing a ramp, the architect must keep the slant or slope gradual enough to easily accommodate a wheelchair. If the slant is too gradual, however, the ramp may become inconveniently long or may require turns to fit it into the available space. The architect will also want to include design features that harmonize with the rest of the building and its surroundings.

Solving a problem such as the design of a ramp often involves writing and determining the solution of several equations or inequalities at the same time.

## 10-1 WRITING AN EQUATION GIVEN SLOPE AND ONE POINT

You have learned to draw a line using the slope and the coordinates of one point on the line. With this same information, we can also write an equation of a line.

**EXAMPLE 1**

Write an equation of the line that has a slope of 4 and that passes through point $(3, 5)$.

*Solution* **METHOD 1.** Use the definition of slope.

*How to Proceed*

(1) Write the definition of slope:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

(2) Let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (x, y)$.
Substitute these values in the definition of slope:

$$4 = \frac{y - 5}{x - 3}$$

(3) Solve the equation for $y$ in terms of $x$:

$$y - 5 = 4(x - 3)$$
$$y - 5 = 4x - 12$$
$$y = 4x - 7$$

**METHOD 2.** Use the slope-intercept form of an equation.

*How to Proceed*

(1) In the equation of a line, $y = mx + b$, replace
$m$ by the given slope, 4:

$$y = mx + b$$
$$y = 4x + b$$

(2) Since the given point, $(3, 5)$, is on the line,
its coordinates satisfy the equation $y = 4x + b$.
Replace $x$ by 3 and $y$ by 5:

$$5 = 4(3) + b$$

(3) Solve the resulting equation to find the value
of $b$, the $y$-intercept:

$$5 = 12 + b$$
$$-7 = b$$

(4) In $y = 4x + b$, replace $b$ by $-7$:

$$y = 4x - 7$$

*Answer* $y = 4x - 7$

**Note:** In standard form $(Ax + By = C)$, the equation of the line is
$4x - y = 7$.

## EXERCISES

### Writing About Mathematics

**1.** Micha says that there is another set of information that can be used to find the equation of a line: the *y*-intercept and the slope of the line. Jen says that that is the same information as the coordinates of one point and the slope of the line. Do you agree with Jen? Explain why or why not.

**2.** In Method 1 used to find the equation of a line with a slope of 4 and that passes through point $(3, 5)$:

    **a.** How can the slope, 4, be written as a ratio?

    **b.** What are the principles used in each step of the solution?

### Developing Skills

In 3–6, in each case, write an equation of the line that has the given slope, *m*, and that passes through the given point.

    **3.** $m = 2; (1, 4)$     **4.** $m = 2; (-3, 4)$     **5.** $m = -3; (-2, -1)$     **6.** $m = -\frac{5}{3}; (-3, 0)$

**7.** Write an equation of the line that has a slope of $\frac{1}{2}$ and a *y*-intercept of 2.

**8.** Write an equation of the line that has a slope of $\frac{-3}{4}$ and a *y*-intercept of 0.

**9.** Write an equation of the line, in the form $y = mx + b$, that is:

    **a.** parallel to the line $y = 2x - 4$, and has a *y*-intercept of 1.

    **b.** perpendicular to the line $y = 2x - 4$, and has a *y*-intercept of 1.

    **c.** parallel to the line $y - 3x = 6$, and has a *y*-intercept of $-2$.

    **d.** perpendicular to the line $y - 3x = 6$, and has a *y*-intercept of $-2$.

    **e.** parallel to the line $2x + 3y = 12$, and passes through the origin.

    **f.** perpendicular to the line $2x + 3y = 12$, and passes through the origin.

**10.** Write an equation of the line, in the form $Ax + By = C$, that is:

    **a.** parallel to the line $y = 4x + 1$, and passes through point $(2, 3)$.

    **b.** perpendicular to the line $y = 4x + 1$, and passes through point $(2, 3)$.

    **c.** parallel to the line $2y - 6x = 9$, and passes through point $(-2, 1)$.

    **d.** perpendicular to the line $2y - 6x = 9$, and passes through point $(-2, 1)$.

    **e.** parallel to the line $y = 4x + 3$, and has the same *y*-intercept as the line $y = 5x - 3$.

    **f.** perpendicular to the line $y = 4x + 3$, and has the same *y*-intercept as the line $y = 5x - 3$.

**11.** For **a**–**c**, write an equation, in the form $y = mx + b$, that describes each graph.

**a.**


**b.**


**c.**
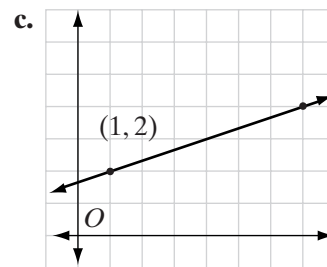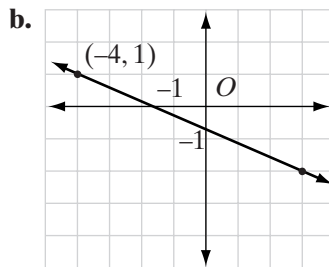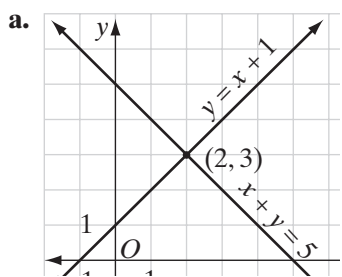

### Applying Skills

**12.** Follow the directions below to draw a map in the coordinate plane.

    **a.** The map begins at the point $A(0, 3)$ with a straight line segment that has a slope of 1. Write an equation for this segment.

    **b.** Point $B$ has an $x$-coordinate of 4 and is a point on the line whose equation you wrote in **a**. Find the coordinates of $B$ and draw $\overline{AB}$.

    **c.** Next, write an equation for a segment from point $B$ that has a slope of $-\frac{1}{2}$.

    **d.** Point $C$ has an $x$-coordinate of 6 and is a point on the line whose equation you wrote in **c**. Find the coordinates of $C$ and draw $\overline{BC}$.

    **e.** Finally, write an equation for a line through $C$ that has a slope of $\frac{1}{2}$.

    **f.** Point $D$ lies on the line whose equation you wrote in **e** and on the $y$-axis. Does point $D$ coincide with point $A$?

**13.** If distance is represented by $y$ and time by $x$, then the rate at which a car travels along a straight road can be represented as slope. Tom leaves his home and drives for 12 miles to the thruway entrance. On the thruway, he travels 65 miles per hour.

    **a.** Write an equation for Tom's distance from his home in terms of the number of hours that he traveled *on the thruway*. (*Hint:* Tom enters the thruway at time 0 when he is at a distance of 12 miles from his home.)

    **b.** How far from home is Tom after 3 hours?

    **c.** How many hours has Tom driven on the thruway when he is 285 miles from home?

## 10-2 WRITING AN EQUATION GIVEN TWO POINTS

You have learned to draw a line using two points on the line. With this same information, we can also write an equation of a line.

**EXAMPLE I**

Write an equation of the line that passes through points $(2, 5)$ and $(4, 11)$.

*Solution* **METHOD I.** Use the definition of slope and the coordinates of the two points to find the slope of the line. Then find the equation of the line using the slope and the coordinates of one point.

*How to Proceed*

(1) Find the slope of the line that passes through the two given points, $(2, 5)$ and $(4, 11)$:

Let $P_1 = (2, 5)$. $\quad [x_1 = 2$ and $y_1 = 5]$
Let $P_2 = (4, 11)$. $\quad [x_2 = 4$ and $y_2 = 11]$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 5}{4 - 2} = \frac{6}{2} = 3$$

(2) In $y = mx + b$, replace $m$ by the slope, 3:

$y = mx + b$
$y = 3x + b$

(3) Select one point that is on the line, for example, $(2, 5)$. Its coordinates must satisfy the equation $y = 3x + b$. Replace $x$ by 2 and $y$ by 5:

$5 = 3(2) + b$

(4) Solve the resulting equation to find the value of $b$, the $y$-intercept:

$5 = 6 + b$
$-1 = b$

(5) In $y = 3x + b$, replace $b$ by $-1$:

$y = 3x - 1$

**METHOD 2.** Let $A(2, 5)$ and $B(4, 11)$ be the two given points on the line and $P(x, y)$ be any point on the line. Use the fact that the slope of $\overline{PA}$ equals the slope of $\overline{AB}$ to write an equation.

*How to Proceed*

(1) Set slope of $\overline{PA}$ equal to slope of $\overline{AB}$:

$$\frac{y - 5}{x - 2} = \frac{5 - 11}{2 - 4}$$

(2) Solve the resulting equation for $y$:

$$\frac{y - 5}{x - 2} = \frac{-6}{-2}$$
$$\frac{y - 5}{x - 2} = 3$$
$$y - 5 = 3(x - 2)$$
$$y - 5 = 3x - 6$$
$$y = 3x - 1$$

*Check* Do the coordinates of the second point, $(4, 11)$, satisfy the equation $y = 3x - 1$?

$$11 \overset{?}{=} 3(4) - 1$$
$$11 = 11 \ ✔$$

*Answer* $y = 3x - 1$

## EXERCISES

### Writing About Mathematics

**1.** In step 3 of Method 1, the coordinates $(2, 5)$ were substituted into the equation. Could the coordinates $(4, 11)$, the coordinates of the other point on the line, be substituted instead? Explain your answer and show that you are correct.

**2.** Name the principle used in each step of the solution of the equation in Method 2.

### Developing Skills

In 3–6, in each case write an equation of the line, in the form $y = mx + b$, that passes through the given points.

**3.** $(0, 5), (-2, 0)$      **4.** $(0, -3), (1, -1)$      **5.** $(1, 4), (3, 8)$      **6.** $(3, 1), (9, 7)$

In 7–10, in each case write an equation of the line, in the form $Ax + By = C$, that passes through the given points.

**7.** $(1, 2), (10, 14)$      **8.** $(0, -1), (6, 8)$      **9.** $(-2, -5), (-1, -2)$      **10.** $(0, 0), (-3, 5)$

**11.** A triangle is determined by the three points $A(3, 5)$, $B(6, 4)$, and $C(1, -1)$. Write the equation of each line in the form $y = mx + b$:

     **a.** $\overleftrightarrow{AB}$          **b.** $\overleftrightarrow{BC}$          **c.** $\overleftrightarrow{CA}$

     **d.** Is triangle $ABC$ a right triangle? Explain your answer.

**12.** A quadrilateral is determined by the four points $W(1, 1)$, $X(-4, 6)$, $Y\left(\frac{1}{2}, 10\frac{1}{2}\right)$, $Z\left(5\frac{3}{5}, 5\frac{3}{5}\right)$. Is the quadrilateral a trapezoid? Explain your answer.

### Applying Skills

**13.** Latonya uploads her digital photos to an internet service that archives them onto CDs for a fee per CD plus a fixed amount for postage and handling; that is, the amount for postage and handling is the same no matter how many archive CDs she purchases. Last month Latonya paid $7.00 for two archive CDs, and this month she paid $13.00 for five archive CDs.

     **a.** Write two ordered pairs such that the $x$-coordinate is the number of archive CDs purchased and the $y$-coordinate is the total cost of archiving the photos.

     **b.** Write an equation for the total cost, $y$, for $x$ archive CDs.

     **c.** What is the domain of the equation found in **b**? Write your answer in set-builder notation.

     **d.** What is the range of the equation found in **b**? Write your answer in set-builder notation.

**14.** Every repair bill at Chickie's Service Spot includes a fixed charge for an estimate of the repairs plus an hourly fee for labor. Jack paid $123 for a TV repair that required 3 hours of labor. Nina paid $65 for a DVD player repair that required 1 hour of labor.

**a.** Write two ordered pairs $(x, y)$, where $x$ represents the number of hours of labor and $y$ is the total cost for repairs.

**b.** Write an equation in the form $y = mx + b$ that expresses the value of $y$, the total cost of repairs, in terms of $x$, the number of hours of labor.

**c.** What is the fixed charge for an estimate at Chickie's Service Spot?

**d.** What is the hourly fee for labor at Chickie's?

**15.** A copying service charges a uniform rate for the first one hundred copies or less and a fee for each additional copy. Nancy Taylor paid $7.00 to make 200 copies and Rosie Barbi paid $9.20 to make 310 copies.

**a.** Write two ordered pairs $(x, y)$, where $x$ represents the number of copies over one hundred and $y$ represents the cost of the copies.

**b.** Write an equation in the form $y = mx + b$ that expresses the value of $y$, the total cost of the copies, in terms of $x$, the number of copies over one hundred.

**c.** What is the cost of the first one hundred copies?

**d.** What is the cost of each additional copy?

## 10-3  WRITING AN EQUATION GIVEN THE INTERCEPTS

The $x$-intercept and the $y$-intercept are two points on the graph of a line. If we know these two points, we can graph the line and we can write the equation of the line. For example, to write the equation of the line whose $x$-intercept is 5 and whose $y$-intercept is $-3$, we can use two points.

- If the $x$-intercept is 5, one point is $(5, 0)$.
- If the $y$-intercept is $-3$, one point is $(0, -3)$.

Let $(x, y)$ be any other point on the line. Set the slope of the line from $(x, y)$ to $(5, 0)$ equal to the slope of the line from $(5, 0)$ to $(0, -3)$.

$$\frac{y - 0}{x - 5} = \frac{0 - (-3)}{5 - 0}$$
$$\frac{y}{x - 5} = \frac{3}{5}$$

Solve the resulting equation for $y$.

$$5y = 3(x - 5)$$
$$5y = 3x - 15$$
$$y = \tfrac{3}{5}x - 3$$

The slope-intercept form of the equation shows that the $y$-intercept is $-3$. Recall from Section 9-6 that we can write this equation in *intercept form* to show both the $x$-intercept and the $y$-intercept.

| | |
|---|---|
| Start with the equation: | $5y = 3x - 15$ |
| Add $-3x$ to each side: | $-3x + 5y = -15$ |
| Divide each term by the constant term: | $\frac{-3x}{-15} + \frac{5y}{-15} = \frac{-15}{-15}$ |
| | $\frac{x}{5} + \frac{y}{-3} = 1$ |

When the equation is in this form, $x$ is divided by the $x$-intercept and $y$ is divided by the $y$-intercept. In other words, when the equation of a line is written in the form $\frac{x}{a} + \frac{y}{b} = 1$, the $x$-intercept is $a$ and the $y$-intercept is $b$.

**EXAMPLE 1**

Write an equation of the line whose $x$-intercept is $-1$ and whose $y$-intercept is $\frac{2}{3}$. Write the answer in standard form ($Ax + By = C$).

**Solution**            *How to Proceed*

(1) In $\frac{x}{a} + \frac{y}{b} = 1$, replace $a$ by $-1$ and $b$ by $\frac{2}{3}$:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-1} + \frac{y}{\frac{2}{3}} = 1$$

(2) Write this equation in simpler form:

$$\frac{x}{-1} + \frac{y}{1} \cdot \frac{3}{2} = 1$$

$$\frac{x}{-1} + \frac{3y}{2} = 1$$

**Note:** In the original form of the equation, $y$ is divided by $\frac{2}{3}$. To divide by $\frac{2}{3}$ is the same as to multiply by the reciprocal, $\frac{3}{2}$.

(3) Write the equation with integral coefficients by multiplying each term of the equation by the product of the denominators, $-2$:

$$-2\left(\frac{x}{-1}\right) - 2\left(\frac{3y}{2}\right) = -2(1)$$

$$2x - 3y = -2$$

*Answer*  $2x - 3y = -2$

**EXAMPLE 2**

Find the $x$-intercept and $y$-intercept of the equation $x + 5y = 2$.

**Solution**   Write the equation in the form $\frac{x}{a} + \frac{y}{b} = 1$. Divide each member of the equation by 2, the constant term:

$$x + 5y = 2$$
$$\frac{x}{2} + \frac{5y}{2} = \frac{2}{2}$$
$$\frac{x}{2} + \frac{5y}{2} = 1$$

The number that divides $x$, 2, is the $x$-intercept. The variable $y$ is multiplied by $\frac{5}{2}$, which is equivalent to saying that it is divided by the reciprocal, $\frac{2}{5}$, so the $y$-intercept is $\frac{2}{5}$.

*Answer*   The $x$-intercept is 2 and the $y$-intercept is $\frac{2}{5}$.

## EXERCISES

### Writing About Mathematics

**1. a.** Can the equation $y = 4$ be put into the form $\frac{x}{a} + \frac{y}{b} = 1$ to find the $x$-intercept and $y$-intercept of the line? Explain your answer.

   **b.** Can the equation $x = 4$ be put into the form $\frac{x}{a} + \frac{y}{b} = 1$ to find the $x$-intercept and $y$-intercept of the line? Explain your answer.

**2.** Can the equation $x + y = 0$ be put into the form $\frac{x}{a} + \frac{y}{b} = 1$ to find the $x$-intercept and $y$-intercept of the line? Explain your answer.
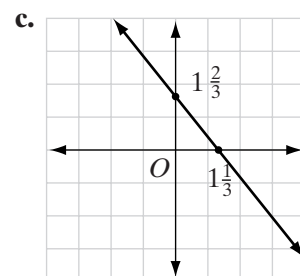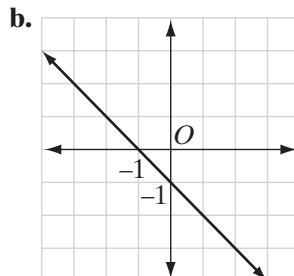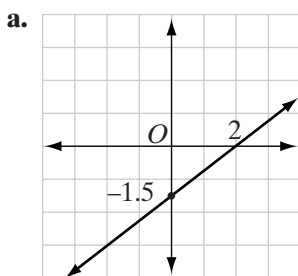
### Developing Skills

In 3–10, find the intercepts of each equation if they exist.

**3.** $x + y = 5$       **4.** $x - 3y = 6$       **5.** $5y = x - 10$       **6.** $2x + y - 2 = 0$

**7.** $3x + 4y = 8$       **8.** $7 - x = 2y$       **9.** $y = 3x - 1$       **10.** $y = -4$

**11.** Express the slope of $\frac{x}{a} + \frac{y}{b} = 1$ in terms of $a$ and $b$.

**12.** For **a–c**, use the $x$- and $y$-intercepts to write an equation for each graph.

**Applying Skills**

**13.** Triangle $ABC$ is drawn on the coordinate plane. Point $A$ is at $(4, 0)$, point $B$ is at $(0, 3)$, and point $C$ is at the origin.

    **a.** What is the equation of $\overleftrightarrow{AB}$ written in the form $\frac{x}{a} + \frac{y}{b} = 1$?

    **b.** What is the length of $\overline{AB}$?

**14. a.** Isosceles right triangle $ABC$ is drawn on the coordinate plane. Write the equation of each side of the triangle if $\angle C$ is the right angle at the origin and the length of each leg is 7.

    **b.** Is more than one answer to **a** possible? Explain your answer.

## 10-4  USING A GRAPH TO SOLVE A SYSTEM OF LINEAR EQUATIONS

### Consistent Equations

The perimeter of a rectangle is 10 feet. When we let $x$ represent the width of the rectangle and $y$ represent the length, the equation $2x + 2y = 10$ expresses the perimeter of the rectangle. This equation, which can be simplified to $x + y = 5$, has infinitely many solutions.

If we also know that the length of the rectangle is 1 foot more than the width, the dimensions of the rectangle can be represented by the equation $y = x + 1$. We want both of the equations, $x + y = 5$ and $y = x + 1$, to be true for the same pair of numbers. The two equations are called a **system of simultaneous equations** or a **linear system**.

> Let $x$ = the width of the rectangle, and $y$ = the length of the rectangle.
>
> Perimeter:
> $$2x + 2y = 10$$
> $$x + y = 5 \qquad \text{(Simplified)}$$
>
> Measures of the sides:
> $$y = x + 1$$
>
> System of simultaneous equations:
> $$x + y = 5$$
> $$y = x + 1$$

The solution of a system of simultaneous equations in two variables is an ordered pair of numbers that satisfies both equations.

The graphs of $x + y = 5$ and $y = x + 1$, drawn using the same set of axes in a coordinate plane, are shown at the right. The possible solutions of $x + y = 5$ are all ordered pairs that are coordinates of the points on the line $x + y = 5$. The possible solutions of $y = x + 1$ are all ordered pairs that are coordinates of the points on the line $y = x + 1$. The coordinates of the point of intersection, $(2, 3)$, are a solution of both equations. The ordered pair $(2, 3)$ is a solution of the system of simultaneous equations.

*Check:* Substitute $(2, 3)$ in both equations:

$$x + y = 5 \qquad\qquad y = x + 1$$
$$2 + 3 \overset{?}{=} 5 \qquad\qquad 3 \overset{?}{=} 2 + 1$$
$$5 = 5 ✔ \qquad\qquad 3 = 3 ✔$$

Since two straight lines can intersect in no more than one point, there is no other ordered pair that is a solution of this system. Therefore $x = 2$ and $y = 3$, or $(2, 3)$, is the solution of the system of equations. The width of the rectangle is 2 feet, and the length of the rectangle is 3 feet.

When two lines are graphed in the same coordinate plane on the same set of axes, one and only one of the following three possibilities can occur. The pair of lines will:

**1.** intersect in one point and have one ordered number pair in common;

**2.** be parallel and have no ordered number pairs in common;

**3.** coincide, that is, be the same line with an infinite number of ordered number pairs in common.

If a system of linear equations such as $x + y = 5$ and $y = x + 1$ has at least one common solution, it is called a **system of consistent equations**. If a system has exactly one solution, it is a **system of independent equations**.
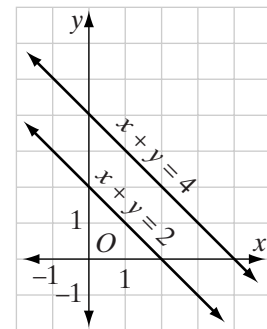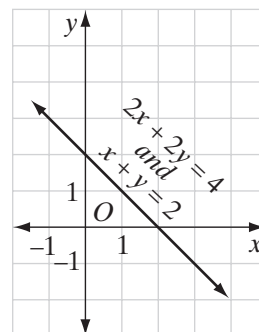
## Inconsistent Equations

Sometimes, as shown at the right, when two linear equations are graphed in a coordinate plane using the same set of axes, the lines are parallel and fail to intersect, as in the case of $x + y = 2$ and $x + y = 4$. There is no common solution for the system of equations $x + y = 2$ and $x + y = 4$. It is obvious that there can be no ordered number pair $(x, y)$ such that the sum of those numbers, $x + y$, is both 2 and 4. Since the solution set of the system has no members, it is the empty set.



If a system of linear equations such as $x + y = 2$ and $x + y = 4$ has no common solution, it is called a **system of inconsistent equations**. The graphs of two inconsistent linear equations are lines that have equal slopes or lines that have no slopes. Such lines are parallel.

## Dependent Equations

Sometimes, as shown at the right, when two linear equations are graphed in a coordinate plane using the same set of axes, the graphs turn out to be the same line; that is, they coincide. This happens in the case of the equations $x + y = 2$ and $2x + 2y = 4$. Every one of the infinite number of solutions of $x + y = 2$ is also a solution of $2x + 2y = 4$. Thus, we see that $2x + 2y = 4$ and $x + y = 2$ are equivalent equations with identical solutions. We note that, when both sides of the equation $2x + 2y = 4$ are divided by 2, the result is $x + y = 2$.



If a system of two linear equations, for example, $x + y = 2$ and $2x + 2y = 4$, is such that every solution of one of the equations is also a solution of the other, it is called a **system of dependent equations**. The graphs of two dependent linear equations are the same line. Note that a system of dependent equations is considered consistent because there is at least one solution.

### Procedure

**To solve a pair of linear equations graphically:**

**1.** Graph one equation in a coordinate plane.

**2.** Graph the other equation using the same set of coordinate axes.

**3.** One of three relationships will apply:

   **a.** If the graphs intersect in one point, the common solution is the ordered pair of numbers that are the coordinates of the point of intersection of the two graphs. The equations are independent and consistent.

   **b.** If the graphs have no points in common, that is, the graphs are parallel, there is no solution. The equations are inconsistent.

   **c.** If the graphs have all points in common, that is, the graphs are the same line, every point on the line is a solution. The equations are consistent and dependent.

**4.** Check any solution by verifying that the ordered pair satisfies both equations.

**EXAMPLE 1**

Solve graphically and check:    $2x + y = 8$
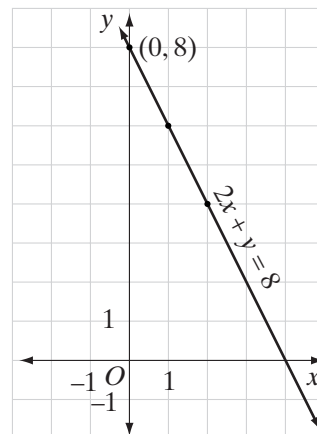$y - x = 2$

***Solution***   (1) Graph the first equation in a coordinate plane.

Solve the first equation for $y$:                               $2x + y = 8$

$$y = -2x + 8$$

The $y$-intercept is 8 and the slope is $-2$. Start at the point $(0, 8)$ and move down 2 units and to the right 1 unit to determine two other points. Or, choose three values of $x$ and find the corresponding values of $y$ to determine three points. Draw the line that is the graph of $2x + y = 8$:

(2) Graph the second equation using the same set of coordinate axes.

Solve the second equation for $y$:                               $y - x = 2$

$$y = x + 2$$

The $y$-intercept is 2 and the slope is 1. Start at the point $(0, 2)$ and move up 1 unit and to the right 1 unit to determine two other points. Or, choose three values of $x$ and find the corresponding values of $y$ to determine three points. Draw the line that is the graph of $y - x = 2$ on the same set of coordinate axes:
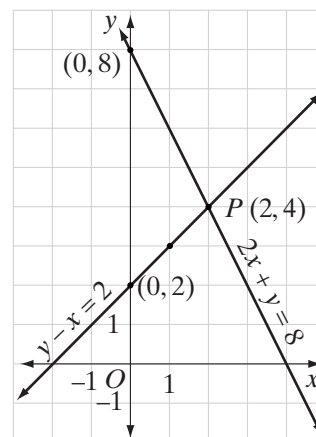
(3) The graphs intersect at one point, $P(2, 4)$.

(4) *Check:* Substitute $(2, 4)$ in each equation:

$$2x + y = 8 \qquad\qquad y - x = 2$$

$$2(2) + 4 \overset{?}{=} 8 \qquad\qquad 4 - 2 \overset{?}{=} 2$$

$$4 + 4 = 8 \; ✔ \qquad\qquad 2 = 2 \; ✔$$
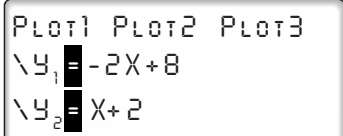
***Calculator*** (1) Solve the equations for $y$:

***Solution***

$$2x + y = 8 \qquad\qquad y - x = 2$$
$$y = -2x + 8 \qquad\qquad y = x + 2$$

(2) Enter the equations in the  Y=  menu.

ENTER:   Y=   (-)   2   X,T,Θ,n   +   8   ENTER

   X,T,Θ,n   +   2

DISPLAY:

```
Plot1 Plot2 Plot3
\Y₁ = -2X+8
\Y₂ = X+2
```

(3) Use ZStandard to display the equations.

ENTER:   ZOOM   6

DISPLAY:
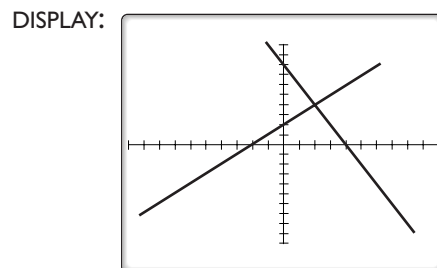


(4) Use  TRACE  to determine the coordinates of the point of intersection.
   Use the right and left arrow keys to move along the first equation to what
   appears to be the point of intersection and note the coordinates, $(2, 4)$.
   Use the up arrow key to change to the second equation. Again note the
   coordinates of the point of intersection, $(2, 4)$.

ENTER:   TRACE

DISPLAY:



```
Y1=-2X+8
```

```
X=2          Y=4
```
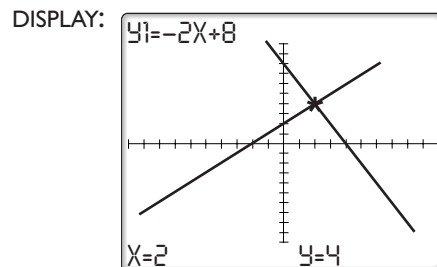
*Answer*   $(2, 4)$, or $x = 2$, $y = 4$, or the solution set $\{(2, 4)\}$

## EXERCISES

### Writing About Mathematics

**1.** Are there any ordered pairs that satisfy both the equations $2x + y = 7$ and $2x = 5 - y$? Explain your answer.

**2.** Are there any ordered pairs that satisfy the equation $y - x = 4$ but do *not* satisfy the equation $2y = 8 + 2x$? Explain your answer.

### Developing Skills

In 3–22, solve each system of equations graphically, and check.

**3.** $y = 2x + 5$
$y = x + 4$

**4.** $y = -2x + 3$
$y = \frac{1}{2}x + 3$

**5.** $x + y = 1$
$x - y = 7$

**6.** $x + y = 5$
$x + 3y = 9$

**7.** $y = 2x + 1$
$x + 2y = 7$

**8.** $x + 2y = 12$
$y = 2x + 6$

**9.** $y = 3x$
$2x + y = 10$

**10.** $x + 3y = 9$
$x = 3$

**11.** $y - x = -2$
$x - 2y = 4$

**12.** $3x + y = 6$
$y = 3$

**13.** $y = 2x + 4$
$x = y - 5$

**14.** $2x - y = 1$
$x = y + 1$

**15.** $y = \frac{1}{3}x - 3$
$2x - y = 8$

**16.** $3x + y = 13$
$x + 6y = -7$

**17.** $2x = y + 9$
$6x + 3y = 15$

**18.** $5x - 3y = 9$
$5y = 13 - x$

**19.** $x = 0$
$y = 0$

**20.** $x + y + 2 = 0$
$x = y - 8$

**21.** $y + 2x + 6 = 0$
$y = x$

**22.** $7x - 4y + 7 = 0$
$3x - 5y + 3 = 0$

In 23–28, in each case: **a.** Graph both equations.   **b.** State whether the system is consistent and independent, consistent and dependent, or inconsistent.

**23.** $x + y = 1$
$x + y = 3$

**24.** $x + y = 5$
$2x + 2y = 10$

**25.** $y = 2x + 1$
$y = 3x + 3$

**26.** $2x - y = 1$
$2y = 4x - 2$

**27.** $y - 3x = 2$
$y = 3x - 2$

**28.** $x + 4y = 6$
$x = 2$

### Applying Skills

In 29–32, in each case: **a.** Write a system of two first-degree equations involving the variables $x$ and $y$ that represent the conditions stated in the problem.   **b.** Solve the system graphically.

**29.** The sum of two numbers is 8. The difference of these numbers is 2. Find the numbers.

**30.** The sum of two numbers is 5. The larger number is 7 more than the smaller number. Find the numbers.

**31.** The perimeter of a rectangle is 12 meters. Its length is twice its width. Find the dimensions of the rectangle.

**32.** The perimeter of a rectangle is 14 centimeters. Its length is 3 centimeters more than its width. Find the length and the width.

**33. a.** The U-Drive-It car rental agency rents cars for $50 a day with unlimited free mileage. Write an equation to show the cost of renting a car from U-Drive-It for one day, $y$, if the car is driven for $x$ miles.

**b.** The Safe Travel car rental agency rents cars for $30 a day plus $0.20 a mile. Write an equation to show the cost of renting a car from Safe Travel for one day, $y$, if the car is driven for $x$ miles.

**c.** Draw, on the same set of axes, the graphs of the equations written in parts **a** and **b**.

**d.** If Greg will drive the car he rents for 200 miles, which agency offers the less expensive car?

**e.** If Sarah will drive the car she rents for 50 miles, which agency offers the less expensive car?

**f.** If Philip finds that the price for both agencies will be the same, how far is he planning to drive the car?

## 10-5  USING ADDITION TO SOLVE A SYSTEM OF LINEAR EQUATIONS

In the preceding section, graphs were used to find solutions of systems of simultaneous equations. Since most of the solutions were integers, the values of $x$ and $y$ were easily read from the graphs. However, it is not always possible to read values accurately from a graph.

For example, the graphs of the system of equations $2x - y = 2$ and $x + y = 2$ are shown at the right. The solution of this system of equations is not a pair of integers. We could approximate the solution and then determine whether our approximation was correct by checking. However, there are other, more direct methods of solution.



Algebraic methods can be used to solve a system of linear equations in two variables. Solutions by these methods often take less time and lead to more accurate results than the graphic method used in Section 4 of this chapter.
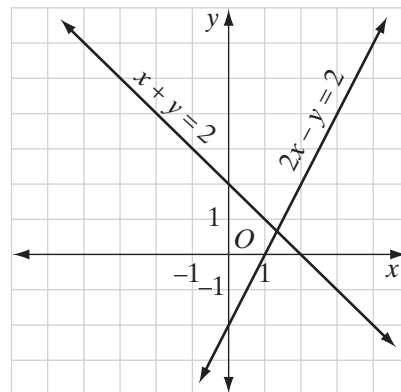
To solve a system of linear equations such as $2x - y = 2$ and $x + y = 2$, we make use of the properties of equality to obtain an equation in one variable. When the coefficient of one of the variables in the first equation is the additive inverse of the coefficient of the same variable in the second, that variable can be eliminated by adding corresponding members of the two equations.

The system $2x - y = 2$ and $x + y = 2$ can be solved by the **addition method** as follows:

Solve for $x$ —
(1) Since the coefficients of $y$ in the two equations are additive inverses, add the equations to obtain an equation that has only one variable:

$$2x - y = 2$$
$$\underline{x + y = 2}$$
$$3x \quad\;\; = 4$$

(2) Solve the resulting equation for $x$:

$$x = \tfrac{4}{3}$$

Solve for $y$ —
(3) Replace $x$ by its value in either of the given equations:

$$x + y = 2$$
$$\tfrac{4}{3} + y = 2$$

(4) Solve the resulting equation for $y$:

$$-\tfrac{4}{3} + \tfrac{4}{3} + y = 2 - \tfrac{4}{3}$$
$$y = \tfrac{2}{3}$$

*Check:* Substitute $\tfrac{4}{3}$ for $x$ and $\tfrac{2}{3}$ for $y$ in each of the given equations, and show that these values make the given equations true:

$$2x - y = 2 \qquad\qquad\qquad x + y = 2$$
$$2\left(\tfrac{4}{3}\right) - \tfrac{2}{3} \overset{?}{=} 2 \qquad\qquad \tfrac{4}{3} + \tfrac{2}{3} \overset{?}{=} 2$$
$$\tfrac{8}{3} - \tfrac{2}{3} \overset{?}{=} 2 \qquad\qquad \tfrac{6}{3} \overset{?}{=} 2$$
$$\tfrac{6}{3} \overset{?}{=} 2 \qquad\qquad\qquad 2 = 2 \checkmark$$
$$2 = 2 \checkmark$$

We were able to add the equations in two variables to obtain an equation in one variable because the coefficients of one of the variables were additive inverses. If the coefficients of neither variable are additive inverses, we can multiply one or both equations by a convenient constant or constants. For clarity, it is often helpful to label the equations in a system. The procedure is shown in the following examples.

**KEEP IN MIND** When solving a system of linear equations for a variable using the addition method, if the result is:

• the equation $0 = 0$, then the system is dependent;

• a false statement (such as $0 = 3$), then the system is inconsistent.

**EXAMPLE 1**

Solve the system of equations and check:

$$x + 3y = 13 \quad [A]$$
$$x + y = 5 \quad [B]$$

*Solution*

*How to Proceed*

(1) Since the coefficients of the variable $x$ are the same in both equations, write an equation equivalent to equation [B] by multiplying both sides of equation [B] by $-1$. Now, since the coefficients of $x$ are additive inverses, add the two equations so that the resulting equation involves one variable, $y$:

$$\begin{array}{rl} x + 3y = 13 & [A] \\ \underline{-x - y = -5} & -1[B] \\ 2y = 8 & \end{array}$$

(2) Solve the resulting equation for the variable $y$:

$$y = 4$$

(3) Replace $y$ by its value in either of the given equations:

$$\begin{array}{rl} x + y = 5 & [B] \\ x + 4 = 5 & \end{array}$$

(4) Solve the resulting equation for the remaining variable, $x$:

$$x = 1$$

*Check* Substitute 1 for $x$ and 4 for $y$ in each of the given equations to verify the solution:

$$x + 3y = 13 \qquad\qquad x + y = 5$$
$$1 + 3(4) \stackrel{?}{=} 13 \qquad\qquad 1 + 4 \stackrel{?}{=} 5$$
$$1 + 12 \stackrel{?}{=} 13 \qquad\qquad 5 = 5 ✔$$
$$13 = 13 ✔$$

*Answer* $x = 1, y = 4$ or $(1, 4)$

**Note:** If equation [A] in Example 1 was $x + y = 13$, then the system of equations would be inconsistent.

$$\begin{array}{rl} x + y = 13 & [A] \\ \underline{-x - y = -5} & -[B] \\ 0 = 8 & ✗ \end{array}$$

**EXAMPLE 2**

Solve the system of equations and check:

$$5a + b = 13 \quad [\text{A}]$$
$$4a - 3b = 18 \quad [\text{B}]$$

**Solution**

*How to Proceed*

(1) Multiply both sides of equation [A] by 3 to obtain an equivalent equation, 3[A]:

$$5a + b = 13 \quad [\text{A}]$$
$$15a + 3b = 39 \quad 3[\text{A}]$$

Note that the coefficient of $b$ in 3[A] is the additive inverse of the coefficient of $b$ in [B].

(2) Add the corresponding members of equations 3[A] and [B] to eliminate the variable $b$:

$$15a + 3b = 39 \quad 3[\text{A}]$$
$$\underline{4a - 3b = 18} \quad [\text{B}]$$
$$19a \quad\quad = 57$$

(3) Solve the resulting equation for $a$:

$$a = 3$$

(4) Replace $a$ by its value in either of the given equations and solve for $b$:

$$5a + b = 13 \quad [\text{A}]$$
$$5(3) + b = 13$$
$$15 + b = 13$$
$$b = -2$$

**Check** Substitute 3 for $a$ and $-2$ for $b$ in the given equations:

$$5a + b = 13 \qquad\qquad 4a - 3b = 18$$
$$5(3) + (-2) \overset{?}{=} 13 \qquad\qquad 4(3) - 3(-2) \overset{?}{=} 18$$
$$15 + (-2) \overset{?}{=} 13 \qquad\qquad 12 + 6 \overset{?}{=} 18$$
$$13 = 13 \checkmark \qquad\qquad 18 = 18 \checkmark$$

**Answer** $a = 3, b = -2$, or $(a, b) = (3, -2)$

Order is critical when expressing solutions. While $(a, b) = (3, -2)$ is the solution to the above system of equations, $(a, b) = (-2, 3)$ is *not*. Be sure that all variables correspond to their correct values in your answers.

**EXAMPLE 3**

Solve and check:

$$7x = 5 - 2y \quad \text{[A]}$$
$$3y = 16 - 2x \quad \text{[B]}$$

**Solution**

*How to Proceed*

(1) Transform each of the given equations into equivalent equations in which the terms containing the variables appear on one side and the constant appears on the other side:

$$7x = 5 - 2y \quad \text{[A]}$$
$$7x + 2y = 5$$
$$3y = 16 - 2x \quad \text{[B]}$$
$$2x + 3y = 16$$

(2) To eliminate $y$, find the least common multiple of the coefficients of $y$ in equations [A] and [B]. That least common multiple is 6. We want to write one equation in which the coefficient of $y$ is 6 and the other in which the coefficient of $y$ is $-6$. Multiply both sides of equation [A] by 3, and multiply both sides of equation [B] by $-2$ so that the new coefficients of $y$ will be additive inverses, 6 and $-6$:

$$3(7x + 2y = 5) \quad 3\text{[A]}$$
$$-2(2x + 3y = 16) \quad -2\text{[B]}$$

(3) Add the corresponding members of this last pair of equations to eliminate variable $y$:

$$21x + 6y = 15 \quad 3\text{[A]}$$
$$\underline{-4x - 6y = -32} \quad -2\text{[B]}$$
$$17x \quad\quad = -17$$

(4) Solve the resulting equation for variable $x$:

$$x = -1$$

(5) Replace $x$ by its value in any equation containing both variables:

$$3y = 16 - 2x \quad \text{[B]}$$
$$3y = 16 - 2(-1)$$
$$3y = 16 + 2$$

(6) Solve the resulting equation for the remaining variable, $y$:

$$3y = 18$$
$$y = 6$$

*Check* Substitute $-1$ for $x$ and 6 for $y$ in each of the original equations to verify the answer. The check is left to you.

*Answer* $x = -1, y = 6$ or $(-1, 6)$

## EXERCISES

### Writing About Mathematics

**1.** Raphael said that the solution to Example 3 could have been found by multiplying equation [A] by $-2$ and equation [B] by 7. Do you agree with Raphael? Explain why or why not.

**2.** Fernando said that the solution to Example 3 could have been found by multiplying equation [A] by $-3$ and equation [B] by 2. Do you agree with Fernando? Explain why or why not.

### Developing Skills

In 3–26, solve each system of equations by using addition to eliminate one of the variables. Check your solution.

**3.** $x + y = 12$
$x - y = 4$

**4.** $a + b = 13$
$a - b = 5$

**5.** $3x + y = 16$
$2x + y = 11$

**6.** $c - 2d = 14$
$c + 3d = 9$

**7.** $a - 4b = -8$
$a - 2b = 0$

**8.** $8a + 5b = 9$
$2a - 5b = -4$

**9.** $-2m + 4n = 13$
$6m + 4n = 9$

**10.** $4x - y = 10$
$2x + 3y = 12$

**11.** $5x + 8y = 1$
$3x + 4y = -1$

**12.** $5x - 2y = 20$
$2x + 3y = 27$

**13.** $2x - y = 26$
$3x - 2y = 42$

**14.** $2x + 3y = 6$
$3x + 5y = 15$

**15.** $5r - 2s = 8$
$3r - 7s = -1$

**16.** $3x + 7y = -2$
$2x + 3y = -3$

**17.** $4x + 3y = -1$
$5x + 4y = 1$

**18.** $4a - 6b = 15$
$6a - 4b = 10$

**19.** $2x + y = 17$
$5x = 25 + y$

**20.** $5r + 3s = 30$
$2r = 12 - 3s$

**21.** $3x - 4y = 2$
$x = 2(7 - y)$

**22.** $3x + 5(y + 2) = 1$
$8y = -3x$

**23.** $\frac{1}{3}x + \frac{1}{4}y = 10$
$\frac{1}{3}x - \frac{1}{2}y = 4$

**24.** $\frac{1}{2}a + \frac{1}{3}b = 8$
$\frac{3}{2}a - \frac{4}{3}b = -4$

**25.** $c - 2d = 1$
$\frac{2}{3}c + 5d = 26$

**26.** $2a = 3b$
$\frac{2}{3}a - \frac{1}{2}b = 2$

### Applying Skills

**27.** Pepe invested $x$ dollars in a savings account and $y$ dollars in a certificate of deposit. His total investment was $500. After 1 year he received 4% interest on the money in his savings account and 6% interest on the certificate of deposit. His total interest was $26. To find how much he invested at each rate, solve the following system of equations:

$$x + y = 500$$
$$0.04x + 0.06y = 26$$

28. Mrs. Briggs deposited a total of $400 in two different banks. One bank paid 3% interest and the other paid 5%. In one year, the total interest was $17. Let $x$ be the amount invested at 3% and $y$ be the amount invested at 5%. To find the amount invested in each bank, solve the following system of equations:

$$x + y = 400$$
$$0.03x + 0.05y = 17$$

29. Heather deposited a total of $600 in two different banks. One bank paid 3% interest and the other 6%. The interest on the account that pays 3% was $9 more than the interest on the account that pays 6%. Let $x$ be the amount invested at 3% and $y$ be the amount invested at 6%. To find the amount Heather invested in each account, solve the following system of equations:

$$x + y = 600$$
$$0.03x = 0.06y + 9$$

30. Greta is twice as old as Robin. In 3 years, Robin will be 4 years younger than Greta is now. Let $g$ represent Greta's current age, and let $r$ represent Robin's current age. To find their current ages, solve the following system of equations:

$$g = 2r$$
$$r + 3 = g - 4$$

31. At the grocery store, Keith buys 3 kiwis and 4 zucchinis for a total of $4.95. A kiwi costs $0.25 more than a zucchini. Let $k$ represent the cost of a kiwi, and let $z$ represent the cost of a zucchini. To find the cost of a kiwi and the cost of a zucchini, solve the following system of equations:

$$3k + 4z = 4.95$$
$$k = z + 0.25$$

32. A 4,000 gallon oil truck is loaded with gasoline and kerosene. The profit on one gallon of gasoline is $0.10 and $0.13 for a gallon of kerosene. Let $g$ represent the number of gallons of gasoline loaded onto the truck and $k$ the number of gallons of kerosene. To find the number of gallons of each fuel that were loaded into the truck when the profit is $430, solve the following system of equations:

$$g + k = 4{,}000$$
$$0.10g + 0.13k = 430$$

## 10-6 USING SUBSTITUTION TO SOLVE A SYSTEM OF LINEAR EQUATIONS

Another algebraic method, called the **substitution method**, can be used to eliminate one of the variables when solving a system of equations. When we use this method, we apply the substitution principle to transform one of the equations of the system into an equivalent equation that involves only one variable.

> **Substitution Principle:** In any statement of equality, a quantity may be sub-stituted for its equal.

To use the substitution method, we must express one variable in terms of the other. Often one of the given equations already expresses one of the variables in terms of the other, as seen in Example 1.

**EXAMPLE 1**

Solve the system of equations and check:

$$4x + 3y = 27 \quad [A]$$
$$y = 2x - 1 \quad [B]$$

*Solution*        *How to Proceed*

(1) Since in equation [B], both $y$ and $2x - 1$ name the same number, eliminate $y$ in equation [A] by replacing it with $2x - 1$:

$$4x + 3y = 27 \qquad [A]$$
$$y = 2x - 1 \qquad [B]$$
$$4x + 3(2x - 1) = 27 \qquad [B{\rightarrow}A]$$

(2) Solve the resulting equation for $x$:

$$4x + 6x - 3 = 27$$
$$10x - 3 = 27$$
$$10x = 30$$
$$x = 3$$

(3) Replace $x$ with its value in any equation involving both variables:

$$y = 2x - 1 \qquad [B]$$
$$y = 2(3) - 1$$

(4) Solve the resulting equation for $y$:

$$y = 6 - 1$$
$$y = 5$$

*Check*  Substitute 3 for $x$ and 5 for $y$ in each of the given equations to verify that the resulting sentences are true.

$$4x + 3y = 27 \qquad\qquad y = 2x - 1$$
$$4(3) + 3(5) \overset{?}{=} 27 \qquad\qquad 5 \overset{?}{=} 2(3) - 1$$
$$12 + 15 \overset{?}{=} 27 \qquad\qquad 5 \overset{?}{=} 6 - 1$$
$$27 = 27 \checkmark \qquad\qquad 5 = 5 \checkmark$$

*Answer*  $x = 3, y = 5$ or $(3, 5)$

**EXAMPLE 2**

Solve the system of equations and check:

$$3x - 4y = 26 \quad \text{[A]}$$
$$x + 2y = 2 \quad \text{[B]}$$

*Solution*  In neither equation is one of the variables expressed in terms of the other. We will use equation [B] in which the coefficient of $x$ is 1 to solve for $x$ in terms of $y$.

*How to Proceed*

(1) Transform one of the equations into an equivalent equation in which one of the variables is expressed in terms of the other. In equation [B], solve for $x$ in terms of $y$:

$$x + 2y = 2$$
$$x = 2 - 2y \quad \text{[B]}$$

(2) Eliminate $x$ from equation [A] by replacing it with its equal, $2 - 2y$, from step (1):

$$3x - 4y = 26 \quad \text{[A]}$$
$$3(2 - 2y) - 4y = 26 \quad \text{[B}\rightarrow\text{A]}$$

(3) Solve the resulting equation for $y$:

$$6 - 6y - 4y = 26$$
$$6 - 10y = 26$$
$$-10y = 20$$
$$y = -2$$

(4) Replace $y$ by its value in any equation that has both variables:

$$x = 2 - 2y \quad \text{[B]}$$
$$x = 2 - 2(-2)$$

(5) Solve the resulting equation for $x$:

$$x = 2 + 4$$
$$x = 6$$

*Check*  Substitute 6 for $x$ and $-2$ for $y$ in each of the given equations to verify that the resulting sentences are true. This check is left to you.

*Answer*  $x = 6, y = -2$ or $(6, -2)$

## EXERCISES

### Writing About Mathematics

**1.** In Example 2, the system of equations could have been solved by first solving the equation $3x - 4y = 26$ for $y$. Explain why the method used in Example 2 was easier.

**2.** Try to solve the system of equations $x - 2y = 5$ and $y = \frac{1}{2}x + 1$ by substituting $\frac{1}{2}x + 1$ for $y$ in the first equation. What conclusion can you draw?

## Developing Skills

In 3–14, solve each system of equations by using substitution to eliminate one of the variables. Check.

**3.** $y = x$
$x + y = 14$

**4.** $y = 2x$
$x + y = 21$

**5.** $a = -2b$
$5a - 3b = 13$

**6.** $y = x + 1$
$x + y = 9$

**7.** $a = 3b + 1$
$5b - 2a = 1$

**8.** $a + b = 11$
$3a - 2b = 8$

**9.** $a - 2b = -2$
$2a - b = 5$

**10.** $7x - 3y = 23$
$x + 2y = 13$

**11.** $4d - 3h = 25$
$3d - 12h = 9$

**12.** $2x = 3y$
$4x - 3y = 12$

**13.** $4y = -3x$
$5x + 8y = 4$

**14.** $2x + 3y = 7$
$4x - 5y = 25$

In 15–26, solve each system of equations by using any convenient algebraic method. Check.

**15.** $s + r = 0$
$r - s = 6$

**16.** $3a - b = 13$
$2a + 3b = 16$

**17.** $y = x - 2$
$3x - y = 16$

**18.** $3x + 8y = 16$
$5x + 10y = 25$

**19.** $y = 3x$
$\frac{1}{3}x + \frac{1}{2}y = 11$

**20.** $a - \frac{2}{3}b = 4$
$\frac{3}{5}a + b = 15$

**21.** $3(y - 6) = 2x$
$3x + 5y = 11$

**22.** $x + y = 300$
$0.1x + 0.3y = 78$

**23.** $3d = 13 - 2c$
$\frac{3c + d}{2} = 8$

**24.** $3x = 4y$
$\frac{3x + 8}{5} = \frac{3y - 1}{2}$

**25.** $\frac{a}{3} = \frac{a + b}{6}$
$5a + 2b = 49$

**26.** $a + 3(b - 1) = 0$
$2(a - 1) + 2b = 16$

## Applying Skills

**27.** The length of the base of an isosceles triangle is 6 centimeters less than the sum of the lengths of the two congruent sides. The perimeter of the triangle is 78 centimeters.

**a.** If $x$ represents the length of each of the congruent sides and $y$ represents the length of the base, express $y$ in terms of $x$.

**b.** Write an equation that represents the perimeter in terms of $x$ and $y$.

**c.** Solve the equations that you wrote in **a** and **b** to find the lengths of the sides of the triangle.

**28.** A package of batteries costs $1.16 more than a roll of film. Martina paid $11.48 for 3 rolls of film and a package of batteries.

**a.** Express the cost of a package of batteries, $y$, in terms of the cost of a roll of film, $x$.

**b.** Write an equation that expresses the amount that Martina paid for the film and batteries in terms of $x$ and $y$.

**c.** Solve the equations that you wrote in **a** and **b** to find what Martina paid for a roll of film and of a package of batteries.

**29.** The cost of the hotdog was $0.30 less than twice the cost of the cola. Jules paid $3.90 for a hotdog and a cola.

    **a.** Express the cost of a hotdog, $y$, in terms of the cost of a cola, $x$.

    **b.** Express what Jules paid for a hotdog and a cola in terms of $x$ and $y$.

    **c.** Solve the equations that you wrote in **a** and **b** to find what Jules paid for a hotdog and for a cola.

**30.** Terri is 12 years older than Jessica. The sum of their ages is 54.

    **a.** Express Terri's age, $y$, in terms of Jessica's age, $x$.

    **b.** Express the sum of their ages in terms of $x$ and $y$.

    **c.** Solve the equations that you wrote in **a** and **b** to find Terri's age and Jessica's age.

## 10-7  USING SYSTEMS OF EQUATIONS TO SOLVE VERBAL PROBLEMS

You have previously learned how to solve word problems by using one variable. Frequently, however, a problem can be solved more easily by using two variables rather than one variable. For example, we can use two variables to solve the following problem:

The sum of two numbers is 8.6. Three times the larger number decreased by twice the smaller is 6.3. What are the numbers?

First, we will represent each number by a different variable:

Let $x$ = the larger number and $y$ = the smaller number.

Now use the conditions of the problem to write two equations:

| The sum of the numbers is 8.6: | $x + y = 8.6$ |
|---|---|
| Three times the larger decreased by twice the smaller is 6.3: | $3x - 2y = 6.3$ |

Solve the system of equations to find the numbers:

$$x + y = 8.6 \quad \rightarrow \quad 2(x + y) = 2(8.6)$$
$$2x + 2y = 17.2$$
$$\underline{3x - 2y = \phantom{0}6.3}$$
$$5x \phantom{+ 2y} = 23.5$$
$$x = \phantom{0}4.7$$

$$x + y = 8.6$$
$$4.7 + y = 8.6$$
$$y = 3.9$$

The two numbers are 4.7 and 3.9.

**Procedure**

**To solve a word problem by using a system of two equations involving two variables:**

**1.** Use two different variables to represent the different unknown quantities in the problem.

**2.** Use formulas or information given in the problem to write a system of two equations in two variables.

**3.** Solve the system of equations.

**4.** Use the solution to determine the answer(s) to the problem.

**5.** Check the answer(s) in the original problem.

**EXAMPLE I**

The owner of a men's clothing store bought six belts and eight hats for $140. A week later, at the same prices, he bought nine belts and six hats for $132. Find the price of a belt and the price of a hat.

***Solution*** (1) Let $b$ = the price, in dollars, of a belt,

and $h$ = the price, in dollars, of a hat.

(2)  6 *belts*  and  8 *hats*  cost  $140.

$$6b + 8h = 140$$

9 *belts*  and  6 *hats*  cost  $132.

$$9b + 6h = 132$$

(3) The least common multiple of the coefficients of $b$ is 18. To eliminate $b$, write equivalent equations with 18 and $-18$ as the coefficients of $b$. Obtain these equations by multiplying both sides of the first equation by 3 and both members of the second equation by $-2$. Then add the equations and solve for $h$.

$$
\begin{aligned}
3(6b + 8h) &= 3(140) &\rightarrow&& 18b + 24h &= 420 \\
-2(9b + 6h) &= -2(132) &\rightarrow&& -18b - 12h &= -264 \\
&&&& 12h &= 156 \\
&&&& h &= 13
\end{aligned}
$$

Substitute 13 for *h* in any equation containing both variables.

$$6b + 8h = 140$$
$$6b + 8(13) = 140$$
$$6b + 104 = 140$$
$$6b = 36$$
$$b = 6$$

(4) *Check:* 6 belts and 8 hats cost $6(\$6) + 8(\$13) = \$36 + \$104 = \$140.$ ✔

9 belts and 6 hats cost $9(\$6) + 6(\$13) = \$54 + \$78 = \$132.$ ✔

*Answer*   A belt costs $6; a hat costs $13.                                    ◰

**EXAMPLE 2**   ▬▬▬▬

When Angelo cashed a check for $170, the bank teller gave him 12 bills, some $20 bills and the rest $10 bills. How many bills of each denomination did Angelo receive?

*Solution*   (1) Represent the unknowns using two variables:

Let *x* = number of $10 bills,

and *y* = number of $20 bills.

In this problem, part of the information is in terms of the *number* of bills (the bank teller gave Angelo 12 bills) and part of the information is in terms of the *value* of the bills (the check was for $170).

(2) Write one equation using the *number* of bills:         $x + y = 12$
Write a second equation using the *value* of the bills.     $10x + 20y = 170$
The value of *x* $10 bills is l0*x*, the value of *y* $20 bills
is 20*y*, and the total value is $170:

(3) Solve the system of equations:

$$\begin{array}{r} 10x + 20y = \quad 170 \\ -10x - 10y = -120 \\ \hline 10y = \quad 50 \\ y = \quad 5 \end{array}$$

$-10(x + y) = -10(12)$  →

$$\begin{array}{r} x + y = 12 \\ x + 5 = 12 \\ x = \ 7 \end{array}$$

(4) *Check* the number of bills:

7 ten-dollar bills and 5 twenty-dollar bills = 12 bills ✔

*Check* the value of the bills:

7 ten-dollar bills are worth $70 and 5 twenty-dollar bills are worth $100.

Total value = $70 + $100 = $170 ✔

*Answer*   Angelo received 7 ten-dollar bills and 5 twenty-dollar bills.                  ◰

## EXERCISES

### Writing About Mathematics

**1.** Midori solved the equations in Example 2 by using the substitution method. Which equation do you think she would have solved for one of the variables? Explain your answer.

**2.** The following system can be solved by drawing a graph, by using the addition method, or by using the substitution method.

$$x + y = 1$$
$$5x + 10y = 8$$

**a.** Which method do you think is the more efficient way of solving this system of equations? Explain why you chose this method.

**b.** Which method do you think is the less efficient way of solving this system of equations? Explain why you chose this method.

### Developing Skills

In 3–9, solve each problem algebraically, using two variables.

**3.** The sum of two numbers is 36. Their difference is 24. Find the numbers.

**4.** The sum of two numbers is 74. The larger number is 3 more than the smaller number. Find the numbers.

**5.** The sum of two numbers is 104. The larger number is 1 less than twice the smaller number. Find the numbers.

**6.** The difference between two numbers is 25. The larger exceeds 3 times the smaller by 4. Find the numbers.

**7.** If 5 times the smaller of two numbers is subtracted from twice the larger, the result is 16. If the larger is increased by 3 times the smaller, the result is 63. Find the numbers.

**8.** One number is 15 more than another. The sum of twice the larger and 3 times the smaller is 182. Find the numbers.

**9.** The sum of two numbers is 900. When 4% of the larger is added to 7% of the smaller, the sum is 48. Find the numbers.

### Applying Skills

In 10–31, solve each problem algebraically using two variables.

**10.** The perimeter of a rectangle is 50 centimeters. The length is 9 centimeters more than the width. Find the length and the width of the rectangle.

**11.** A rectangle has a perimeter of 38 feet. The length is 1 foot less than 3 times the width. Find the dimensions of the rectangle.

**12.** Two angles are supplementary. The larger angle measures 120° more than the smaller. Find the degree measure of each angle.

13. Two angles are supplementary. The larger angle measures 15° less than twice the smaller. Find the degree measure of each angle.

14. Two angles are complementary. The measure of the larger angle is 30° more than the measure of the smaller angle. Find the degree measure of each angle.

15. The measure of the larger of two complementary angles is 6° less than twice the measure of the smaller angle. Find the degree measure of each angle.

16. In an isosceles triangle, each base angle measures 30° more than the vertex angle. Find the degree measures of the three angles of the triangle.

17. At a snack bar, 3 pretzels and 1 can of soda cost $2.75. Two pretzels and 1 can of soda cost $2.00. Find the cost of a pretzel and the cost of a can of soda.

18. On one day, 4 gardeners and 4 helpers earned $360. On another day, working the same number of hours and at the same rate of pay, 5 gardeners and 6 helpers earned $480. Each gardener receives the same pay for a day's work and each helper receives the same pay for a day's work. How much does a gardener and how much does a helper earn each day?

19. A baseball manager bought 4 bats and 9 balls for $76.50. On another day, she bought 3 bats and 1 dozen balls at the same prices and paid $81.00. How much did she pay for each bat and each ball?

20. Mrs. Black bought 2 pounds of veal and 3 pounds of pork, for which she paid $20.00. Mr. Cook, paying the same prices, paid $11.25 for 1 pound of veal and 2 pounds of pork. Find the price of a pound of veal and the price of a pound of pork.

21. One day, Mrs. Rubero paid $18.70 for 4 kilograms of brown rice and 3 kilograms of basmati rice. Next day, Mrs. Leung paid $13.30 for 3 kilograms of brown rice and 2 kilograms of basmati rice. If the prices were the same on each day, find the price per kilogram for each type of rice.

22. Tickets for a high school dance cost $10 each if purchased in advance of the dance, but $15 each if bought at the door. If 100 tickets were sold and $1,200 was collected, how many tickets were sold in advance and how many were sold at the door?

23. A dealer sold 200 tennis racquets. Some were sold for $33 each, and the rest were sold on sale for $18 each. The total receipts from these sales were $4,800. How many racquets did the dealer sell at $18 each?

24. Mrs. Rinaldo changed a $100 bill in a bank. She received $20 bills and $10 bills. The number of $20 bills was 2 more than the number of $10 bills. How many bills of each kind did she receive?

25. Linda spent $4.50 for stamps to mail packages. Some were 39-cent stamps and the rest were 24-cent stamps. The number of 39-cent stamps was 3 less than the number of 24-cent stamps. How many stamps of each kind did Linda buy?

26. At the Savemore Supermarket, 3 pounds of squash and 2 pounds of eggplant cost $2.85. The cost of 4 pounds of squash and 5 pounds of eggplant is $5.41. What is the cost of one pound of squash, and what is the cost of one pound of eggplant?

27. One year, Roger Jackson and his wife Wilma together earned $67,000. If Roger earned $4,000 more than Wilma earned that year, how much did each earn?

**28.** Mrs. Moto invested $1,400, part at 5% and part at 8%. Her total annual income from both investments was $100. Find the amount she invested at each rate.

**29.** Mr. Stein invested a sum of money in certificates of deposit yielding 4% a year and another sum in bonds yielding 6% a year. In all, he invested $4,000. If his total annual income from the two investments was $188, how much did he invest at each rate?

**30.** Mr. May invested $21,000, part at 8% and the rest at 6%. If the annual incomes from both investments were equal, find the amount invested at each rate.

**31.** A dealer has some hard candy worth $2.00 a pound and some worth $3.00 a pound. He makes a mixture of these candies worth $45.00. If he used 10 pounds more of the less expensive candy than he used of the more expensive candy, how many pounds of each kind did he use?

## 10-8 GRAPHING THE SOLUTION SET OF A SYSTEM OF INEQUALITIES

To find the solution set of a **system of inequalities**, we must find the ordered pairs that satisfy the open sentences of the system. We do this by a graphic procedure that is similar to the method used in finding the solution set of a system of equations.

### EXAMPLE 1

Graph the solution set of this system:    $x > 2$
$y < -2$

***Solution*** (1) Graph $x > 2$ by first graphing the plane divider $x = 2$, which, in the figure at the right, is represented by the dashed line labeled $l$. The half-plane to the right of this line is the graph of the solution set of $x > 2$.
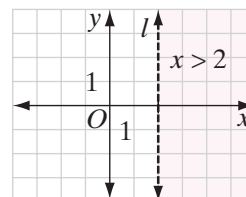


(2) Using the same set of axes, graph $y < -2$ by first graphing the plane divider $y = -2$, which, in the figure at the right, is represented by the dashed line labeled $m$. The half-plane below this line is the graph of the solution set of $y < -2$.
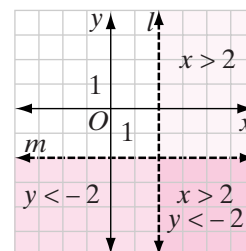


(3) The solution set of the system $x > 2$ and $y < -2$ consists of the intersection of the solution sets of $x > 2$ and $y < -2$. Therefore, the dark colored region in the lower figure, which is the intersection of the graphs made in steps (1) and (2), is the graph of the solution set of the system $x > 2$ and $y < -2$.

From the graph on page 431, all points in the solution region, and no others, satisfy both inequalities of the system. For example, point $(4, -3)$, which lies in the region, satisfies the system because its $x$-value satisfies one of the given inequalities, $4 > 2$, and its $y$-value satisfies the other inequality, $-3 < -2$.

**EXAMPLE 2**

Graph the solution set of $3 < x < 5$ in a coordinate plane.

**Solution**    The inequality $3 < x < 5$ means $3 < x$ and $x < 5$. This may be written as $x > 3$ and $x < 5$.

(1) Graph $x > 3$ by first graphing the plane divider $x = 3$, which, in the figure at the right, is represented by the dashed line labeled $l$. The half-plane to the right of line $x = 3$ is the graph of the solution set of $x > 3$.
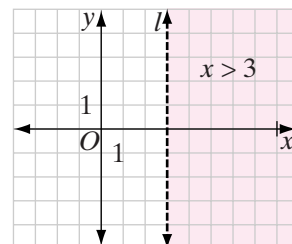
(2) Using the same set of axes, graph $x < 5$ by first graphing the plane divider $x = 5$, which, in the figure at the right, is represented by the dashed line labeled $m$. The half-plane to the left of line $x = 5$ is the graph of the solution set of $x < 5$.
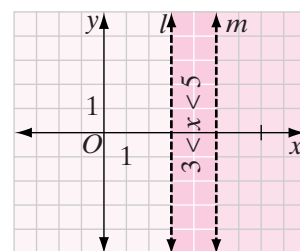
(3) The dark colored region, which is the intersection of the graphs made in steps (1) and (2), is the graph of the solution set of $3 < x < 5$, or $x > 3$ and $x < 5$. All points in this region, and no others, satisfy $3 < x < 5$. For example, point $(4, 3)$, which lies in the region and whose $x$-value is 4, satisfies $3 < x < 5$ because $3 < 4 < 5$ is a true statement.

**EXAMPLE 3**

Graph the following system of inequalities and label the solution set $R$:

$$x + y \geq 4$$
$$y \leq 2x - 3$$

**Solution**    (1) Graph $x + y \geq 4$ by first graphing the plane divider $x + y = 4$, which, in the figure at the top of page 433, is represented by the solid line labeled $l$. The line $x + y = 4$ and the half-plane above this line together form the graph of the solution set of $x + y \geq 4$.

(2) Using the same set of axes, graph $y \leq 2x - 3$ by first graphing the plane divider $y = 2x - 3$. In the figure at the right, this is the solid line labeled $m$. The line $y = 2x - 3$ and the half-plane below this line together form the graph of the solution set of $y \leq 2x - 3$.

(3) The dark colored region, the intersection of the graphs made in steps (1) and (2), is the solution set of the system $x + y \geq 4$ and $y \leq 2x - 3$. Label it $R$. Any point in the region $R$, such as $(5, 2)$, will satisfy $x + y \geq 4$ because $5 + 2 \geq 4$, or $7 \geq 4$, is true, and will satisfy $y \leq 2x - 3$ because $2 \leq 2(5) - 3$, or $2 \leq 7$, is true.
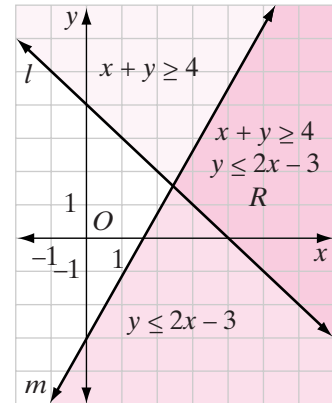


## EXAMPLE 4

Sandi boards cats and dogs while their owners are away. Each week she can care for no more than 12 animals. For next week she already has reservations for 4 cats and 5 dogs, but she knows those numbers will probably increase. Draw a graph to show the possible numbers of cats and dogs that Sandi might board next week and list all possible numbers of cats and dogs.

***Solution***

(1) Let $x =$ the number of cats Sandi will board, and $y =$ the number of dogs.

(2) Use the information in the problem to write inequalities.
Sandi can care for no more than 12 animals: $\qquad x + y \leq 12$
She expects at least 4 cats: $\qquad x \geq 4$
She expects at least 5 dogs: $\qquad y \geq 5$

(3) Draw the graphs of these three inequalities.

(4) The possible numbers of cats and dogs are represented by the ordered pairs of positive integers that are included in the solution set of the inequalities. These ordered pairs are shown as points on the graph.

The points $(4, 5)$, $(4, 6)$, $(4, 7)$, $(4, 8)$, $(5, 5)$, $(5, 6)$, $(5, 7)$, $(6, 5)$, $(6, 6)$, $(7, 5)$, all satisfy the given conditions.
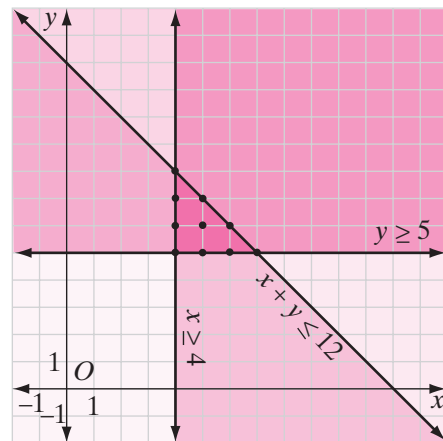
*Answer*  If there are 4 cats, there can be 5, 6, 7, or 8 dogs.

If there are 5 cats, there can be 5, 6, or 7 dogs.

If there are 6 cats, there can be 5 or 6 dogs.

If there are 7 cats, there can be 5 dogs.

## EXERCISES

### Writing About Mathematics

**1.** Write a system of inequalities whose solution set is the unshaded region of the graph drawn in Example 3. Explain your answer.

**2.** What points on the graph drawn in Example 3 are in the solution set of the system of open sentences $x + y \geq 4$ and $y = 2x - 3$? Explain your answer.

**3.** Describe the solution set of the system of inequalities $y > 4x$ and $4x - y \geq 3$. Explain why this occurs.

### Developing Skills

In 4–18, graph each system of inequalities and label the solution set $S$. Check one solution.

| | | |
|---|---|---|
| **4.** $x \geq 1$ <br> $y > -2$ | **5.** $x < 0$ <br> $y > 0$ | **6.** $y \geq 1$ <br> $y < x - 1$ |
| **7.** $y > x$ <br> $y < 2x + 3$ | **8.** $y \geq 2x$ <br> $y > x + 3$ | **9.** $y \leq 2x + 3$ <br> $y \geq -x$ |
| **10.** $y - x \geq 5$ <br> $y - 2x \leq 7$ | **11.** $y < x - 1$ <br> $x + y \geq 2$ | **12.** $y + 3x \geq 6$ <br> $y < 2x - 4$ |
| **13.** $x + y > 3$ <br> $x - y < 6$ | **14.** $x - y \leq -2$ <br> $x + y \geq 2$ | **15.** $2x + y \leq 6$ <br> $x + y - 2 > 0$ |
| **16.** $2x + 3y \geq 6$ <br> $x + y - 4 \leq 0$ | **17.** $y \geq \frac{1}{2}x$ <br> $x > 0$ | **18.** $x + \frac{3}{4}y \leq 3$ <br> $\frac{1}{3}y - \frac{2}{3}x < 0$ |

In 19–23, in each case, graph the solution set in a coordinate plane.
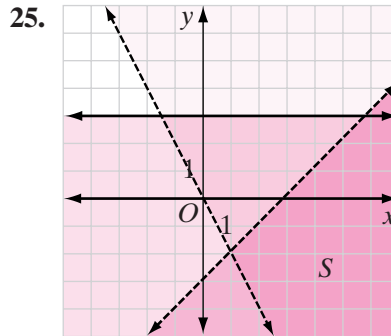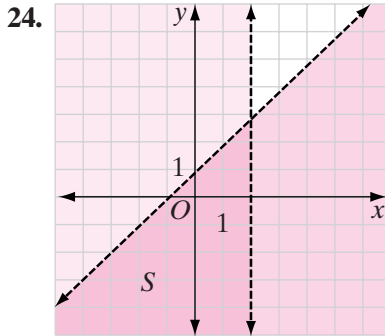
**19.** $1 < x < 4$          **20.** $-5 \leq x \leq -1$          **21.** $2 < y \leq 6$

**22.** $-2 \leq y \leq 3$          **23.** $(y \geq -1)$ and $(y < 5)$

In 24 and 25, write the system of equation whose solution set is labeled *S*.

**24.**



**25.**



## Applying Skills

**26.** In Ms. Dwyer's class, the number of boys is more than twice the number of girls. There are at least 2 girls. There are no more than 12 boys.

   **a.** Write the three sentences given above as three inequalities, letting $x$ equal the number of girls and $y$ equal the number of boys.

   **b.** On one set of axes, graph the three inequalities written in part **a**.

   **c.** Label the solution set of the system of inequalities *S*.

   **d.** Do the coordinates of every point in the region labeled *S* represent the possible number of girls and number of boys in Ms. Dwyer's class? Explain your answer.

   **e.** Write one ordered pair that could represent the number of girls and boys in Ms. Dwyer's class.

**27.** When Mr. Ehmke drives to work, he drives on city streets for part of the trip and on the expressway for the rest of the trip. The total trip is less than 8 miles. He drives at least 1 mile on city streets and at least 2 miles on the expressway.

   **a.** Write three inequalities to represent the information given above, letting $x$ equal the number of miles driven on city streets and $y$ equal the number of miles driven on the expressway.

   **b.** On one set of axes, graph the three inequalities written in part **a**.

   **c.** Label the solution set of the system of inequalities *R*.

   **d.** Do the coordinates of every point in the region labeled *R* represent the possible number of miles driven on city streets and driven on the expressway? Explain your answer.

   **e.** Write one ordered pair that could represent the number of miles Mr. Ehmke drives on city streets and on the expressway.

**28.** Mildred bakes cakes and pies for sale. She takes orders for the baked goods but also makes extras to sell in her shop. On any day, she can make a total of no more than 12 cakes and pies. For Monday, she had orders for 4 cakes and 6 pies. Draw a graph and list the possible number of cakes and pies she might make on Monday.

## CHAPTER SUMMARY

The equation of a line can be written if one of the following sets of information is known:

**1.** The slope and one point

**2.** The slope and the $y$-intercept

**3.** Two points

**4.** The $x$-intercept and the $y$-intercept

A system of two linear equations in two variables may be:

**1. Consistent:** its solution is at least one ordered pair of numbers. (The graphs of the equations are intersecting lines or lines that coincide.)

**2. Inconsistent:** its solution is the empty set. (The graphs of the equations are parallel with no points in common.)

**3. Dependent:** its solution is an infinite set of number pairs. (The graphs of the equations are the same line.)

**4. Independent:** its solution set is exactly one ordered pair of numbers.

The solution of an independent linear system in two variables may be found **graphically** by determining the coordinates of the point of intersection of the graphs or **algebraically** by using **addition** or **substitution**. Systems of linear equations can be used to solve verbal problems.

The solution set of a system of inequalities can be shown on a graph as the **intersection** of the solution sets of the inequalities.

## VOCABULARY

**10-4** System of simultaneous equations • Linear system • System of consistent equations • System of independent equations • System of inconsistent equations • System of dependent equations

**10-5** Addition method

**10-6** Substitution method • Substitution principle

**10-8** System of inequalities

## REVIEW EXERCISES

**1.** Solve the following system of equations for $x$ and $y$ and check the solution:

$$5x - 2y = 22$$
$$x + 2y = 2$$

**2. a.** Graph the following system of inequalities and check one solution:
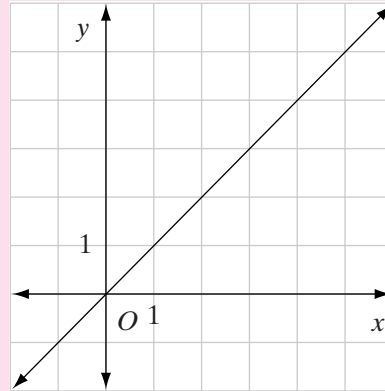
$$y \le 2x + 3$$
$$y \le 2x$$

    **b.** Describe the solution set of the system in terms of the solution sets of the individual inequalities.
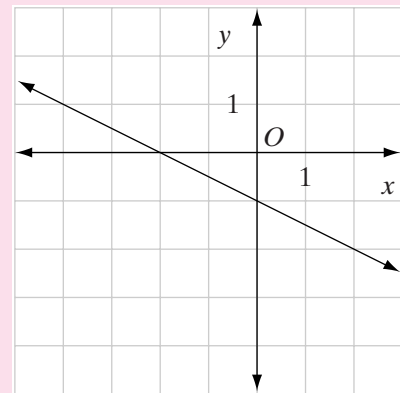
In 3–8, write the equation of the line, in the form $y = mx + b$, that satisfies each of the given conditions.

   **3.** Through $(1, -2)$ with slope 3

   **4.** Through $(5, 6)$ with slope $-1\frac{1}{2}$

   **5.** Through $(-2, 5)$ and $(2, -3)$

   **6.** Through $(-1, -3)$ and $(5, -3)$

   **7.** With slope $2\frac{2}{3}$ and $y$-intercept 0

   **8.** Graphed to the right

In 9–14, write the equation of the line, in the form $\frac{x}{a} + \frac{y}{b} = 1$, that satisfies each of the given conditions.

   **9.** With slope 1 and $y$-intercept $-1$

  **10.** With slope $-3.5$ and $y$-intercept $-1.5$

  **11.** With $x$-intercept 5 and $y$-intercept $-2$

  **12.** With $x$-intercept $\frac{3}{4}$ and $y$-intercept $-\frac{5}{7}$

  **13.** Through $(1, 3)$ and $(5, 6)$

  **14.** Graphed to the right

In 15–17, write the equation of the line, in the form $Ax + By = C$, that satisfies each of the given conditions.

  **15.** Parallel to the $y$-axis and 3 units to the left of the $y$-axis

  **16.** Through $(0, 4)$ and $(2, 0)$

  **17.** Through $(1, 1)$ and $(4, 5)$

In 18–20, solve each system of equations *graphically* and check.

  **18.** $x + y = 6$           **19.** $y = -x$           **20.** $2y = x + 4$

      $y = 2x - 6$            $2x + y = 3$           $x - y + 4 = 0$

In 21–23, solve each system of equations by using *addition* to eliminate one of the variables. Check.

**21.** $2x + y = 10$
$\quad x + y = 3$

**22.** $x + 4y = 1$
$\quad 5x - 6y = -8$

**23.** $3c + d = 0$
$\quad c - 4d = 52$

In 24–26, solve each system of equations by using *substitution* to eliminate one of the variables. Check.

**24** $x + 2y = 7$
$\quad x = y - 8$

**25.** $3r + 2s = 20$
$\quad r = -2s$

**26.** $x + y = 7$
$\quad 2x + 3y = 21$

In 27–32, solve each system of equations by using an appropriate algebraic method. Check.

**27.** $x + y = 0$
$\quad 3x + 2y = 5$

**28.** $5a + 3b = 17$
$\quad 4a - 5b = 21$

**29.** $t + u = 12$
$\quad t = \frac{1}{3}u$

**30.** $3a = 4b$
$\quad 4a - 5b = 2$

**31.** $x + y = 1{,}000$
$\quad 0.06x = 0.04y$

**32.** $10t + u = 24$
$\quad t + u = \frac{1}{7}(10u + t)$

**33. a.** Solve this system of equations algebraically:

$$x - y = 3$$
$$x + 3y = 9$$

**b.** On a set of coordinate axes, graph the system of equations given in part **a**.

In 34–36, graph each system of inequalities and label the solution set $A$.

**34.** $y > 2x - 3$
$\quad y \le 5 - x$

**35.** $y \le \frac{1}{2}x$
$\quad x \ge -4$

**36.** $2x + y < 4$
$\quad x - y < -2$

In 37–41, for each problem, write two equations in two variables and solve algebraically.

**37.** The sum of two numbers is 7. Their difference is 18. Find the numbers.

**38.** At a store, 3 notebooks and 2 pencils cost $2.80. At the same prices, 2 notebooks and 5 pencils cost $2.60. Find the cost of a notebook and of a pencil.

**39.** Two angles are complementary. The larger angle measures 15° less than twice the smaller angle. Find the degree measure of each angle.

**40.** The measure of the vertex angle of an isosceles triangle is 3 times the measure of each of the base angles. Find the measure of each angle of the triangle.

**41.** At the beginning of the year, Lourdes's salary increased by 3%. If her weekly salary is now $254.41, what was her weekly salary last year?

### Exploration

Jason and his brother Robby live 1.25 miles from school. Jason rides his bicycle to school and Robby walks. The graph below shows their relative positions on their way to school one morning.

Use the information provided by the graph to describe Robby's and Jason's journeys to school. Who left first? Who arrived at school first? If each boy took the same route to school, when did Jason pass Robby? How fast did Robby walk? How fast did Jason ride his bicycle?
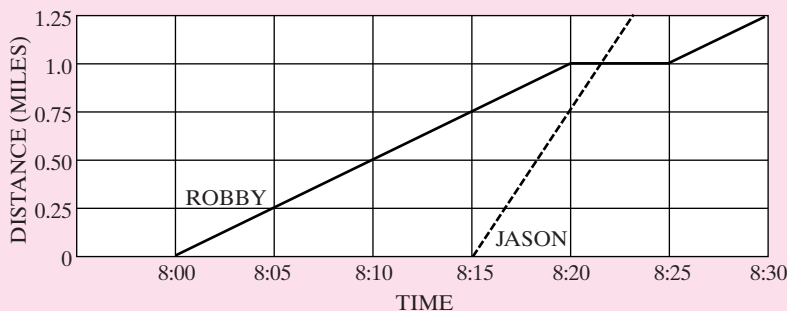


# CUMULATIVE REVIEW                    CHAPTERS 1–10

## Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

**1.** The multiplicative inverse of $-0.4$ is
(1) 0.4      (2) 1      (3) 2.5      (4) $-2.5$

**2.** Which of the following numbers has the greatest value?
(1) $-1.5$      (2) $-1.\overline{5}$      (3) $-\sqrt{2}$      (4) $-0.5\pi$

**3.** Which of the following is an example of the associative property of addition?
(1) $6 + (x + 3) = 6 + (3 + x)$      (3) $3(4a) = (3 \times 4)a$
(2) $2(x + 2) = 2x + 4$      (4) $3 + (4 + a) = (3 + 4) + a$

**4.** Which of the following is a point on the line whose equation is $x - y = -5$?
(1) $(-1, -4)$      (2) $(1, -4)$      (3) $(-1, 4)$      (4) $(1, 4)$

**5.** The expression $5 - 2(3x^2 + 8)$ is equivalent to
(1) $9x^2 + 24$      (2) $-6x^2 - 21$      (3) $9x^2 + 8$      (4) $-6x^2 - 11$

**6.** The solution of the equation $0.2x - 3 = x + 1$ is
(1) $-5$      (2) 5      (3) 0.5      (4) 50

**7.** The product of $3.40 \times 10^{-3}$ and $8.50 \times 10^2$ equals
(1) $2.89 \times 10^0$        (2) $2.89 \times 10^1$        (3) $2.89 \times 10^2$        (4) $2.89 \times 10^{-1}$

**8.** The measure of $\angle A$ is 12° less than twice the measure of its complement. What is the measure of $\angle A$?
(1) 51°                (2) 34°                (3) 39°                (4) 56°

**9.** The $y$-intercept of the graph of $2x + 3y = 6$ is
(1) 6                (2) 2                (3) 3                (4) −3

**10.** When $2a^2 - 5a$ is subtracted from $5a^2 + 1$, the difference is
(1) $-3a^2 - 5a - 1$                        (3) $3a^2 + 6a$
(2) $3a^2 - 5a + 1$                        (4) $3a^2 + 5a + 1$

## Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

**11.** A straight line has a slope of $-3$ and contains the point $(0, 6)$. What are the coordinates of the point at which the line intersects the $x$-axis?

**12.** Last year the Edwards family spent $6,200 for food. This year, the cost of food for the family was $6,355. What was the percent of increase in the cost of food for the Edwards family?

## Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

**13.** Dana paid $3.30 for 2 muffins and a cup of coffee. At the same prices, Damion paid $5.35 for 3 muffins and 2 cups of coffee. What is the cost of a muffin and of a cup of coffee?

**14.** In trapezoid $ABCD$, $\overline{BC} \perp \overline{AB}$ and $\overline{BC} \perp \overline{DC}$. If $AB = 54.8$ feet, $DC = 37.2$ feet, and $BC = 15.8$ feet, find to the *nearest degree* the measure of $\angle A$.

## Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

**15.** Tamara wants to develop her own pictures but does not own photographic equipment. The Community Darkroom makes the use of photographic equipment available for a fee. Membership costs $25 a month and members pay $1.50 an hour for the use of the equipment. Non-members may also use the equipment for $4.00 an hour.

    **a.** Draw a graph that shows the cost of becoming a member and using the photographic equipment. Let the vertical axis represent cost in dollars and the horizontal axis represent hours of use.

    **b.** On the same graph, show the cost of using the photographic equipment for non-members.

    **c.** When Tamara became a member of the Community Darkroom in June, she found that her cost for that month would have been the same if she had not been a member. How many hours did Tamara use the facilities of the darkroom in June?

**16.** $ABCD$ is a rectangle, $E$ is a point on $\overline{DC}$, $AD = 24$ centimeters, $DE = 32$ centimeters, and $EC = 10$ centimeters.

    **a.** Find the area of $\triangle ABE$.

    **b.** Find the perimeter of $\triangle ABE$.