CHAPTER

5

OPERATIONS WITH ALGEBRAIC EXPRESSIONS

Marvin is planning two rectangular gardens that will have the same width. He wants one to be 5 feet longer than it is wide and the other to be 8 feet longer than it is wide. How can he express the area of each of the gardens and the total area of the two gardens in terms of w, the width of each?

Problems like this often occur in many areas of business, science and technology as well as every day life. When we use variables and the rules for adding and multiplying expressions involving variables, we can often write general expressions that help us investigate many possibilities in the solution of a problem. In this chapter, you will learn to add, subtract, multiply, and divide algebraic expressions.

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5-1 ADDING AND SUBTRACTING ALGEBRAIC EXPRESSIONS

Recall that an algebraic expression that is a number, a variable, or a product or quotient of numbers and variables is called a **term**. Examples of terms are:

$$7 \qquad a \qquad -2b \qquad -\tfrac{4}{7}y^2 \qquad 0.7ab^5 \qquad -\tfrac{5}{w}$$

Two or more terms that contain the same variable or variables with corresponding variables having the same exponents, are called **like terms** or **similar terms**. For example, the following pairs are like terms.

$$6k \text{ and } k \qquad 5x^2 \text{ and } -7x^2 \qquad 9ab \text{ and } 0.4ab \qquad \tfrac{9}{2}x^2y^3 \text{ and } -\tfrac{11}{3}x^2y^3$$

Two terms are **unlike terms** when they contain different variables, or the same variable or variables with different exponents. For example, the following pairs are unlike terms.

$$3x \text{ and } 4y \qquad 5x^2 \text{ and } 5x^3 \qquad 9ab \text{ and } 0.4a \qquad \tfrac{8}{3}x^3y^2 \text{ and } \tfrac{4}{7}x^2y^3$$

To add or subtract like terms, we use the distributive property of multiplication over addition or subtraction.

$$9x + 2x = (9 + 2)x = 11x$$
$$-16cd + 3cd = (-16 + 3)cd = -13cd$$
$$18y^2 - 5y^2 = (18 - 5)y^2 = 13y^2$$
$$7ab - ab = 7ab - 1ab = (7 - 1)ab = 6ab$$

Since the distributive property is true for any number of terms, we can express the sum or difference of any number of like terms as a single term.

$$-3ab^2 + 4ab^2 - 2ab^2 = (-3 + 4 - 2)ab^2 = -1\,ab^2 = -\,ab^2$$
$$x^3 + 11x^3 - 8x^3 - 4x^3 = (1 + 11 - 8 - 4)x^3 = 0x^3 = 0$$

Recall that when like terms are added:

1. The sum or difference has the same variable or variables as the original terms.
2. The numerical coefficient of the sum or difference is the sum or difference of the numerical coefficients of the terms that were added.

The sum of unlike terms cannot be expressed as a single term. For example, the sum of $2x$ and $3y$ cannot be written as a single term but is written $2x + 3y$.

EXAMPLE 1

Add: *Answers*

a. $+3a + (-8a)$ $= [3 + (-8)]a = -5a$

b. $-12b^2 - (-5b^2)$ $= [-12 - (-5)]b^2 = [-12 + 5]b^2 = -7b^2$

c. $-15abc + 6abc$ $= (-15 + 6)abc = -9abc$

d. $8x^2y - x^2y$ $= (8 - 1)x^2y = 7x^2y$

e. $-9y + 9y$ $= (-9 + 9)y = 0y = 0$

f. $2(a + b) + 6(a + b)$ $= (2 + 6)(a + b) = 8(a + b)$

EXAMPLE 2

An isosceles triangle has two sides that are equal in length. The length of each of the two equal sides of an isosceles triangle is twice the length of the third side of the triangle. If the length of the third side is represented by n, represent in simplest form the perimeter of the triangle.

Solution n represents the length of the base.

$2n$ represents the length of one of the equal sides.

$2n$ represents the length of the other equal side.

$$\text{Perimeter} = n + 2n + 2n = (1 + 2 + 2)n = 5n.$$

Note that the length of a side of a geometric figure is a positive number. Therefore, the variable n must represent a positive real number, that is, the replacement set for n must be the set of positive real numbers.

Answer $5n$

Monomials and Polynomials

A term that has no variable in the denominator is called a **monomial**. For example, 5, $-5w$, and $\frac{3w^2}{5}$ are monomials, but $\frac{5}{w}$ is *not* a monomial.

A monomial or the sum of monomials is called a **polynomial**. A polynomial may have one or more terms. Some polynomials are given special names to indicate the number of terms.

- A monomial such as $4x^2$ may be considered to be a polynomial of one term. (*Mono-* means "one"; *poly-* means "many.")
- A polynomial of two unlike terms, such as $10a + 12b$, is called a **binomial**. (*Bi-* means "two.")

- A polynomial of three unlike terms, such as $x^2 + 3x + 2$, is called a **trinomial**. (*Tri-* means "three.")
- A polynomial such as $5x^2 + (-2x) + (-4)$ is usually written as $5x^2 - 2x - 4$.

A polynomial has been simplified or is in **simplest form** when it contains no like terms. For example, $5x^3 + 8x^2 - 5x^3 + 7$, when expressed in simplest form, becomes $8x^2 + 7$.

A polynomial is said to be in **descending order** when the exponents of a particular variable decrease as we move from left to right. The polynomial $x^3 + 5x^2 - 4x + 9$ is in a descending order of powers of x.

A polynomial is said to be in **ascending order** when the exponents of a particular variable increase as we move from left to right. The polynomial $4 + 5y + y^2$ is in an ascending order of powers of y.

To add two polynomials, we use the commutative, associative, and distributive properties to combine like terms.

EXAMPLE 3

Simplify: $3ab + 5b - ab + 4ab - 2b$

Solution

How to Proceed	
(1) Write the expression:	$3ab + 5b - ab + 4ab - 2b$
(2) Group like terms together by using the commutative and associative properties:	$3ab - ab + 4ab + 5b - 2b$ $(3ab - ab + 4ab) + (5b - 2b)$
(3) Use the distributive property:	$(3 - 1 + 4)ab + (5 - 2)b$
(4) Simplify the numerical expressions that are in parentheses:	$6ab + 3b$

Answer $6ab + 3b$

EXAMPLE 4

Find the sum: $(3x^2 + 5) + (6x^2 + 8)$

Solution

How to Proceed	
(1) Write the expression:	$(3x^2 + 5) + (6x^2 + 8)$
(2) Use the associative property:	$3x^2 + (5 + 6x^2) + 8$
(3) Use the commutative property:	$3x^2 + (6x^2 + 5) + 8$
(4) Use the associative property:	$(3x^2 + 6x^2) + (5 + 8)$
(5) Add like terms:	$9x^2 + 13$

Answer $9x^2 + 13$

The sum of polynomials can also be arranged vertically, placing like terms under one another. The sum of $3x^2 + 5$ and $6x^2 + 8$ can be arranged as shown at the right.

Addition can be checked by substituting any convenient value for the variable and evaluating each polynomial and the sum.

$$\begin{array}{l} 3x^2 + 5 \\ \underline{6x^2 + 8} \\ 9x^2 + 13 \end{array}$$

Check Let $x = 4$

$$\begin{array}{l} 3x^2 + 5 = 3(4)^2 + 5 = 53 \\ \underline{6x^2 + 8} = \underline{6(4)^2 + 8 = 104} \\ 9x^2 + 13 = 9(4)^2 + 13 = 157 \checkmark \end{array}$$

EXAMPLE 5

Simplify: $6a + [5a + (6 - 3a)]$

Solution When one grouping symbol appears within another, first simplify the expression within the innermost grouping symbol.

How to Proceed

(1) Write the expression	$6a + [5a + (6 - 3a)]$
(2) Use the commutative property:	$6a + [5a + (-3a + 6)]$
(3) Use the associative property:	$6a + [(5a - 3a) + 6]$
(4) Combine like terms:	$6a + [2a + 6]$
(5) Use the associative property:	$(6a + 2a) + 6$
(6) Combine like terms:	$8a + 6$

Answer $8a + 6$

EXAMPLE 6

Express the difference $(4x^2 + 2x - 3) - (2x^2 - 5x - 3)$ in simplest form.

Solution

How to Proceed

(1) Write the subtraction problem:	$(4x^2 + 2x - 3) - (2x^2 - 5x - 3)$
(2) To subtract, add the opposite of the polynomial to be subtracted:	$(4x^2 + 2x - 3) + (-2x^2 + 5x + 3)$
(3) Use the commutative and associative properties to group like terms:	$(4x^2 - 2x^2) + (2x + 5x) + (-3 + 3)$
(4) Add like terms:	$2x^2 + 7x + 0$ $2x^2 + 7x$

Answer $2x^2 + 7x$

EXERCISES

Writing About Mathematics

1. Christopher said that $3x - x = 3$.
   a. Use the distributive property to show Christopher that his answer is incorrect.
   b. Substitute a numerical value of x to show Christopher that his answer is incorrect.

2. Explain how the procedure for adding like terms is similar to the procedure for adding fractions.

Developing Skills

In 3–27, write each algebraic expression in simplest form.

3. $(+8c) + (+7c)$
4. $(-4a) + (-6a)$
5. $(-20r) + (5r)$
6. $(-7w) + (+7w)$
7. $(5ab) + (-9ab)$
8. $(+6x) + (-4x) + (-5x) + (10x)$
9. $-5y + 6y + 9y - 14y$
10. $4m + 9m - 12m - m$
11. $(+8x^2) + (-x^2) + (-12x^2) + (+2x^2)$
12. $4a + (9a + 3)$
13. $7b + (4b - 6)$
14. $8c + (7 - 9c)$
15. $(-6x - 4) + 5x$
16. $r + (s + 2r)$
17. $8d^2 + (6d^2 - 4d)$
18. $(5x + 3) - (6x - 5)$
19. $-9y + [7 + (6y - 7)]$
20. $(5 - 6y) + (-9y + 2)$
21. $5a + [3b - (-2a + 4b)]$
22. $(5x^2 - 4) + (-3x^2 - 9)$
23. $3y^2 + [6y^2 + (3y - 4)]$
24. $(x^3 + 3x^2) - (-2x^2 - 9)$
25. $-d^2 + [9d + (2 - 4d^2)]$
26. $(x^2 + 5x - 24) + (-x^2 - 4x + 9)$
27. $(x^3 + 9x - 5) - (-4x^2 - 12x - 5)$

In 28–31, state whether each expression is a monomial, a binomial, a trinomial, or none of these.

28. $8x + 3$
29. $7y$
30. $-2a^2 + 3a - 6$
31. $x^3 + 2x^2 + x - 7$

32. a. Give an example of the sum of two binomials that is a binomial.
    b. Give an example of the sum of two binomials that is a monomial.
    c. Give an example of the sum of two binomials that is a trinomial.
    d. Give an example of the sum of two binomials that has four terms.
    e. Can the sum of two binomials have more than four terms?

Applying Skills

In 33–41, write each answer as a polynomial in simplest form.

33. A cheeseburger costs 3 times as much as a soft drink, and an order of fries costs twice as much as a soft drink. If a soft drink costs s cents, express the total cost of a cheeseburger, an order of fries, and a soft drink in terms of s.

34. Jack deposited some money in his savings account in September. In October he deposited twice as much as in September, and in November he deposited one-half as much as in September. If x represents the amount of money deposited in September, represent, in terms of x, the total amount Jack deposited in the 3 months.

35. On Tuesday, Melita read 3 times as many pages as she read on Monday. On Wednesday she read 1.5 times as many pages as on Monday, and on Thursday she read half as many pages as on Monday. If Melita read p pages on Monday, represent in terms of p, the total number of pages she read in the 4 days.

36. The cost of 12 gallons of gas is represented by $12x$, and the cost of a quart of oil is represented by $2x - 30$. Represent the cost of 12 gallons of gas and a quart of oil.

37. In the last basketball game of the season, Tom scored $2x$ points, Tony scored $x + 5$ points, Walt scored $3x + 1$ points, Dick scored $4x - 7$ points, and Dan scored $2x - 2$ points. Represent the total points scored by these five players.

38. Last week, Greg spent twice as much on bus fare as he did on lunch, and 3 dollars less on entertainment than he did on bus fare. If x represents the amount, in dollars, spent on lunch, express in terms of x the total amount Greg spent on lunch, bus fare, and entertainment.

39. The cost of a chocolate shake is 40 cents less than the cost of a hamburger. If h represents the cost, *in cents*, of a hamburger, represent in terms of h the cost of a hamburger and a chocolate shake *in dollars*.

40. Rosie spent 12 dollars more for fabric for a new dress than she did for buttons, and 1 dollar less for thread than she did for buttons. If b represents the cost, in dollars, of the buttons, represent in terms of b the total cost of the materials needed for the dress.

41. The length of a rectangle is $7z^2 + 3$ inches and the width is $9z^2 + 2$ inches. Represent the perimeter of the rectangle.

5-2 MULTIPLYING POWERS THAT HAVE THE SAME BASE

Finding the Product of Powers

We know that y^2 means $y \cdot y$ and y^3 means $y \cdot y \cdot y$. Therefore,

$$y^2 \cdot y^3 = \overbrace{(y \cdot y)}^{2}\overbrace{(y \cdot y \cdot y)}^{3} = \overbrace{(y \cdot y \cdot y \cdot y \cdot y)}^{5} = y^5$$

Similarly,

$$c^2 \cdot c^4 = \overbrace{(c \cdot c)}^{2}\overbrace{(c \cdot c \cdot c \cdot c)}^{4} = \overbrace{(c \cdot c \cdot c \cdot c \cdot c \cdot c)}^{6} = c^6$$

and

$$x \cdot x^3 = \overbrace{(x)}^{1}\overbrace{(x \cdot x \cdot x)}^{3} = \overbrace{(x \cdot x \cdot x \cdot x)}^{4} = x^4$$

The exponent in each product is the sum of the exponents in the factors, as shown in these examples.

In general, when x is a real number and a and b are positive integers:

$$x^a \cdot x^b = x^{a+b}$$

EXAMPLE 1

Simplify each of the following products:

a. $x^5 \cdot x^2$ **b.** $a^7 \cdot a$ **c.** $3^2 \cdot 3^4$

Answers **a.** $x^5 \cdot x^2 = x^{5+2} = x^7$ **b.** $a^7 \cdot a = a^{7+1} = a^8$ **c.** $3^2 \cdot 3^4 = 3^{2+4} = 3^6$

Note: When we multiply powers with like bases, we do not actually perform the operation of multiplication but rather *count up* the number of times that the base is to be used as a factor to find the product. In Example 1c above, the answer does not give the value of the product but indicates only the number of times that 3 must be used as a factor to obtain the product. We can use the power key [^] to evaluate the products $3^2 \cdot 3^4$ and 3^6 to show that they are equal.

ENTER: 3 [^] 2 [×] 3 [^] 4 [ENTER]

DISPLAY:
```
3^2*3^4
            729
```

ENTER: 3 [^] 6 [ENTER]

DISPLAY:
```
3^6
            729
```

Finding a Power of a Power

Since $(x^3)^4 = x^3 \cdot x^3 \cdot x^3 \cdot x^3$, then $(x^3)^4 = x^{12}$. The exponent 12 can be obtained by addition: $3 + 3 + 3 + 3 = 12$ or by multiplication: $4 \times 3 = 12$.

In general, when x is a real number and a and c are positive integers:

$$(x^a)^c = x^{ac}$$

An expression such as $(x^5y^2)^3$ can be simplified by using the commutative and associative properties:

$$\begin{aligned}(x^5y^2)^3 &= (x^5y^2)(x^5y^2)(x^5y^2)\\ &= (x^5 \cdot x^5 \cdot x^5)(y^2 \cdot y^2 \cdot y^2)\\ &= x^{15}y^6\end{aligned}$$

When the base is the product of two or more factors, we apply the rule for the power of a power to each factor.

$$(x^5y^2)^3 = (x^5)^3\,(y^2)^3 = x^{5(3)}y^{2(3)} = x^{15}y^6$$

Thus,

$$\mathbf{(x^ay^b)^c = (x^a)^c(y^b)^c = x^{ac}y^{bc}}$$

The expression $(5 \cdot 4)^3$ can be evaluated in two ways.

$$(5 \cdot 4)^3 = 5^3 \cdot 4^3 = 125 \cdot 64 = 8{,}000$$
$$(5 \cdot 4)^3 = 20^3 = 8{,}000$$

EXAMPLE 2

Simplify each expression in two ways.

a. $(a^2)^3$ **b.** $(ab^2)^4$ **c.** $(3^2 \cdot 4^2)^3$

Solution **a.**

$$(a^2)^3 = a^2 \cdot a^2 \cdot a^2 = a^{2+2+2} = a^6$$

or

$$(a^2)^3 = a^{2(3)} = a^6$$

Answer a^6

b.

$$(ab^2)^4 = ab^2 \cdot ab^2 \cdot ab^2 \cdot ab^2 = (a \cdot a \cdot a \cdot a)(b^2 \cdot b^2 \cdot b^2 \cdot b^2) = a^4b^8$$

or

$$(ab^2)^4 = a^{1(4)}b^{2(4)} = a^4b^8$$

Answer a^4b^8

c.

$$(3^2 \cdot 4^2)^3 = (3^2)^3 \cdot (4^2)^3 = 3^6 \cdot 4^6 = (3 \cdot 4)^6 = 12^6$$

or

$$(3^2 \cdot 4^2)^3 = ((3 \cdot 4)^2)^3 = (12^2)^3 = 12^6$$

Answer 12^6

To evaluate the expression in Example 2c, use a calculator.

Evaluate $(3^2 \cdot 4^2)^3$:

ENTER: (3 x^2 × 4 x^2) ^ 3 ENTER

DISPLAY:
```
(3²*4²)^3
            2985984
```

Evaluate $(12)^6$:

ENTER: 12 ^ 6 ENTER

DISPLAY:
```
12^6
            2985984
```

EXERCISES

Writing About Mathematics

1. Does $-5^3 \cdot 5^3 = -25^3$? Use the commutative and associative properties of multiplication to explain why or why not.
2. Does $-2^4 \cdot 4 = -2^6$? Use the commutative and associative properties of multiplication to explain why or why not.

Developing Skills

In 3–26, multiply in each case.

3. $a^2 \cdot a^3$ **4.** $b^3 \cdot b^4$ **5.** $r^2 \cdot r^4 \cdot r^5$ **6.** $r^3 \cdot r^3$

7. $z^3 \cdot z^3 \cdot z^5$ **8.** $t^8 \cdot t^4 \cdot t^2$ **9.** $x \cdot x$ **10.** $a^2 \cdot a$

11. $e^4 \cdot e^5 \cdot e$ **12.** $2^3 \cdot 2^2$ **13.** $3^4 \cdot 3^3$ **14.** $5^2 \cdot 5^4$

15. $4^3 \cdot 4$ **16.** $2^4 \cdot 2^5 \cdot 2$ **17.** $(x^3)^2$ **18.** $(a^4)^2$

19. $(z^3)^2 \cdot (z^4)^2$ **20.** $(x^2y^3)^2$ **21.** $(ab^2)^4$ **22.** $(rs)^3$

23. $(2^2 \cdot 3^2)^3$ **24.** $(5 \cdot 2^3)^4$ **25.** $(100^2 \cdot 10^3)^5$ **26.** $(a^2)^5 \cdot a$

In 27–31, multiply in each case. (All exponents are positive integers.)

27. $x^a \cdot x^{2a}$ **28.** $y^c \cdot y^2$ **29.** $c^r \cdot c^2$ **30.** $x^m \cdot x$ **31.** $(3y)^a \cdot (3y)^b$

In 32–39, state whether each sentence is true or false.

32. $10^4 \cdot 10^3 = 10^7$ **33.** $2^4 \cdot 2^2 = 2^8$ **34.** $3^3 \cdot 2^2 = 6^5$ **35.** $14^{80} \cdot 14^{10} = 14^{90}$

36. $3^3 \cdot 2^2 = 6^6$ **37.** $5^4 \cdot 5 = 5^5$ **38.** $(2^2)^3 = 2^5$ **39.** $(6^3)^4 = (6^4)^3$

Applying Skills

40. Two students attended the first meeting of the Chess Club. At that meeting, they decided that each person would bring one additional person to the next meeting, doubling the membership. At the second meeting, they again decided that each person would bring one additional person to the next meeting, again doubling the membership. If this plan was carried out for n meetings, the membership would equal 2^n persons.

a. How many persons attended the fifth meeting?

b. At which meeting would the membership be twice as large as at the fifth meeting?

41. In the metric system, 1 meter = 10^2 centimeters and 1 kilometer = 10^3 meters. How many centimeters equal one kilometer?

5-3 MULTIPLYING BY A MONOMIAL

Multiplying a Monomial by a Monomial

We know that the commutative property of multiplication makes it possible to arrange the factors of a product in any order and that the associative property of multiplication makes it possible to group the factors in any combination. For example:

$$(5x)(6y) = (5)(6)(x)(y) = (5 \cdot 6)(x \cdot y) = 30xy$$
$$(3x)(7x) = (3)(7)(x)(x) = (3 \cdot 7)\ (x \cdot x) = 21x^2$$
$$(-2x^2)(+5x^4) = (-2)(x^2)(+5)(x^4) = [(-2)(+5)]\ [(x^2)(x^4)] = -10x^6$$
$$(-3a^2b^3)(-4a^4b) = (-3)(a^2)\ (b^3)(-4)(a^4)\ (b)$$
$$= [(-3)(-4)][(a^2)(a^4)][(b^3)(b)] = 12a^6b^4$$

In the preceding examples, the factors may be rearranged and grouped mentally.

Procedure

To multiply a monomial by a monomial:

1. Use the commutative and associative properties to rearrange and group the factors. This may be done mentally.
2. Multiply the numerical coefficients.
3. Multiply powers with the same base by adding exponents.
4. Multiply the products obtained in Steps 2 and 3 and any other variable factors by writing them with no sign between them.

EXAMPLE 1

Multiply:

	Answers		*Answers*
a. $(8xy)(3z)$	$= 24xyz$	**b.** $(-4a^3)(-5a^5)$	$= 20a^8$
c. $(-6y^3)(y)$	$= -6y^4$	**d.** $(3a^2b^3)(4a^3b^4)$	$= 12a^5b^7$
e. $(-5x^2y^3)(-2xy^2)$	$= 10x^3y^5$	**f.** $(6c^2d^4)(-0.5d)$	$= -3c^2d^5$
g. $(-3x^2)^3$	$= (-3x^2)(-3x^2)(-3x^2) = -27x^6$		
	or		
	$= (-3)^3(x^2)^3 = -27x^6$		

EXAMPLE 2

Represent the area of a rectangle whose length is $3x$ and whose width is $2x$.

Solution

How to Proceed

(1) Write the area formula:	$A = lw$
(2) Substitute the values of l and w:	$= (3x)(2x)$
(3) Perform the multiplication:	$= (3 \cdot 2)(x \cdot x)$
	$= 6x^2$

Answer $6x^2$

Multiplying a Polynomial by a Monomial

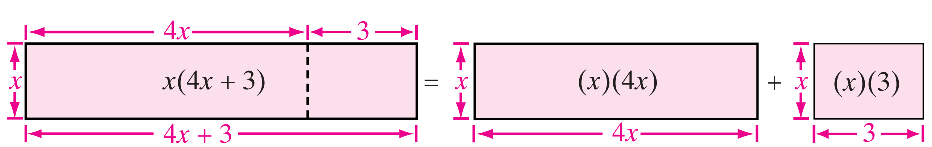
The distributive property of multiplication over addition is used to multiply a polynomial by a monomial. Therefore,

$$a(b + c) = ab + ac$$

$$x(4x + 3) = x(4x) + x(3)$$

$$= 4x^2 + 3x$$

This result can be illustrated geometrically. Let us separate a rectangle, whose length is $4x + 3$ and whose width is x, into two smaller rectangles such that the length of one rectangle is $4x$ and the length of the other is 3.



Since the area of the largest rectangle is equal to the sum of the areas of the two smaller rectangles:

$$x(4x + 3) = x(4x) + x(3) = 4x^2 + 3x$$

Procedure

To multiply a polynomial by monomial, use the distributive property: Multiply each term of the polynomial by the monomial and write the result as the sum of these products.

Multiplication and Grouping Symbols

When an algebraic expression involves grouping symbols such as parentheses, we follow the general order of operations and perform operations with algebraic terms.

In the example at the right, first simplify the expression within parentheses:	$8y - 2(7y - 4y) + 5$ $8y - 2(3y) + 5$
Next, multiply:	$8y - 6y + 5$
Finally, combine like terms by addition or subtraction:	$2y + 5$

In many expressions, however, the terms within parentheses cannot be combined because they are unlike terms. When this happens, we use the distributive property to clear parentheses and then follow the order of multiplying before adding.

Here, clear the parentheses by using the distributive property:	$3 + 7(2x + 3)$ $3 + 7(2x) + 7(3)$
Next, multiply:	$3 + 14x + 21$
Finally, combine like terms by addition:	$24 + 14x$

The multiplicative identity property states that $a = 1 \cdot a$. By using this property, we can say that $5 + (2x - 3) = 5 + 1(2x - 3)$ and then follow the procedures shown above.

$$5 + (2x - 3) = 5 + 1(2x - 3) = 5 + 1(2x) + 1(-3) = 5 + 2x - 3 = 2 + 2x$$

Also, since $-a = -1 \cdot a$, we can use this property to simplify expressions in which a parentheses is preceded by a negative sign:

$$6y - (9 - 7y) = 6y - 1(9 - 7y) = 6y - 1(9) - 1(-7y) = 6y - 9 + 7y = 13y - 9$$

EXAMPLE 3

Multiply:

	Answers
a. $5(r - 7)$	$= 5r - 35$
b. $8(3x - 2y + 4z)$	$= 24x - 16y + 32z$
c. $-5x(x^2 - 2x + 4)$	$= -5x^3 + 10x^2 - 20x$
d. $-3a^2b^2(4ab^2 - 3b^2)$	$= -12a^3b^4 + 9a^2b^4$

EXAMPLE 4

Simplify: **a.** $-5x(x^2 - 2) - 7x$ **b.** $3a - (5 - 7a)$

a. *How to Proceed*

(1) Write the expression:	$-5x(x^2 - 2) - 7x$
(2) Use the distributive property:	$-5x(x^2) - 5x(-2) - 7x$
(3) Multiply:	$-5x^3 + 10x - 7x$
(4) Add like terms:	$-5x^3 + 3x$

Answer $-5x^3 + 3x$

b. *How to Proceed*

(1) Write the expression:	$3a - (5 - 7a)$
	$3a - 1(5 - 7a)$
(2) Use the distributive property:	$3a - 5 + 7a$
(3) Add like terms:	$10a - 5$

Answer $10a - 5$

EXERCISES

Writing About Mathematics

1. In an algebraic term, how do you show the product of a constant times a variable or the product of different variables?

2. In the expression $2 + 3(7y)$, which operation is performed first? Explain your answer.

3. In the expression $(2 + 3)(7y)$, which operation is performed first? Explain your answer.

4. In the expression $5y(y + 3)$, which operation is performed first? Explain your answer.

5. Can the sum $x^2 + x^3$ be written in simpler form? Explain your answer.

6. Can the product $x^2(x^3)$ be written in simpler form? Explain your answer.

Developing Skills

In 7–29, find each product.

7. $(-4b)(-6b)$ **8.** $(+5)(-2y)(-3y)$ **9.** $(4a)(5b)$

10. $(-8r)(-2r)$ **11.** $(+7x)(-2y)(3z)$ **12.** $(+6x)(-0.5y)$

13. $\left(-\frac{3}{4}a\right)\left(+8b\right)$ **14.** $\left(-6x\right)\left(\frac{1}{2}y\right)\left(-\frac{1}{3}z\right)$ **15.** $(+5ab)(-3c)$

16. $(-7r)(5st)$ **17.** $(-2)(+6cd)(-e)$ **18.** $(+9xy)(-2x)$

19. $(3s)(-4s)(5t)$ **20.** $(+5a^2)(-4a^2)$ **21.** $(-6x^4)(-3x^3)$

22. $(20y^3))(-7y^2)$ **23.** $(18r^5)(-5r^2)$ **24.** $(+3z^2)(4z)$

25. $(-8y^5)(5y)$ **26.** $(-9z)(8z^4)(z^3)$ **27.** $(+6x^2y^3)(-4x^4y^2)$

28. $(-7a^3b)(+5a^2b^2)$ **29.** $(+4ab^2)(-2a^2b^3)$

In 30–47, write each product as a polynomial.

30. $3(6c + 3d)$ **31.** $-5(4m - 6n)$ **32.** $-2(8a + 6b)$

33. $10(2x - 0.2y)$ **34.** $12\left(\frac{2}{3}m - 4n\right)$ **35.** $-8\left(4r - \frac{1}{4}s\right)$

36. $-16\left(\frac{3}{4}c - \frac{5}{8}d\right)$ **37.** $4x(5x + 6)$ **38.** $5d(d^2 - 3d)$

39. $-5c^2(15c - 4c)$ **40.** $mn(m + n)$ **41.** $-ab(a - b)$

42. $3ab(5a^2 - 7b^2)$ **43.** $-r^3s^3(-2r^4s - 3s^4)$ **44.** $10d(2a - 3c + 4b)$

45. $-8(2x^2 - 3x - 5)$ **46.** $3xy(x^2 + xy + y^2)$ **47.** $5r^2s^2(-2r^2 + 3rs - 4s^2)$

In 48–50, represent the area of each rectangle whose length l and width w are given.

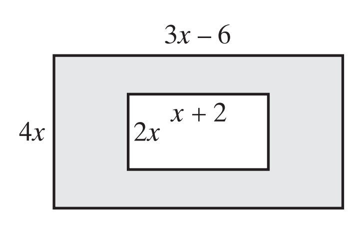
48. $l = 5y, w = 3y$ **49.** $l = 3x, w = 5y$ **50.** $l = 3c, w = 8c - 2$

51. The dimensions of the outer rectangle pictured at the right are $4x$ by $3x - 6$. The dimensions of the inner rectangle are $2x$ by $x + 2$.

a. Express the area of the outer rectangle in terms of x.

b. Express the area of the inner rectangle in terms of x.

c. Express as a polynomial in simplest form the area of the shaded region.



In 52–73, simplify each expression.

52. $5(d + 3) - 10$ **53.** $3(2 - 3c) + 5c$ **54.** $7 + 2(7x - 5)$

55. $2(x - 1) + 6$ **56.** $4(3 - 6a - 8s)$ **57.** $5 - 4(3e - 5)$

58. $8 + (4e - 2)$

59. $a + (b - a)$

60. $(6b + 4) - 2b$

61. $9 - (5t + 6)$

62. $4 - (2 - 8s)$

63. $-(6x - 7) + 14$

64. $5x(2x - 3) + 9x$

65. $12y - 3y(2y - 4)$

66. $7x + 3(2x - 1) - 8$

67. $7c - 4d - 2(4c - 3d)$

68. $3a - 2a(5a - a) + a^2$

69. $(a + 3b) - (a - 3b)$

70. $4(2x + 5) - 3(2 - 7x)$

71. $3(x + y) + 2(x - 3y)$

72. $5x(2 - 3x) - x(3x - 1)$

73. $y(y + 4) - y(y - 3) - 9y$

Applying Skills

In 74–86, write each answer as a polynomial in simplest form.

74. If 1 pound of grass seed costs $25x$ cents, represent in terms of x the cost of 7 pounds of seed.

75. If a bus travels at the rate of $10z$ miles per hour for 4 hours, represent in terms of z the distance traveled.

76. If Lois has $2n$ nickels, represent in terms of n the number of cents she has.

77. If the cost of a notebook is $2x - 3$, express the cost of five notebooks.

78. If the length of a rectangle is $5y - 7$ and the width is $3y$, represent the area of the rectangle.

79. If the measure of the base of a triangle is $3b + 2$ and the height is $4b$, represent the area of the triangle.

80. Represent the distance traveled in 3 hours by a car traveling at $3x - 7$ miles per hour.

81. Represent in terms of x and y the amount saved in $3y$ weeks if $x - 2$ dollars are saved each week.

82. The length of a rectangular skating rink is 2 less than 3 times the width. If w represents the width of the rink, represent the area in terms of w.

83. An internet bookshop lists used books for $3x - 5$ dollars each. The cost for shipping and handling is 2 dollars for five books or fewer. Represent the total cost of an order for four used books.

84. A store advertises skirts for $x - 5$ dollars and allows an additional 10-dollar reduction on the total purchase if three or more skirts are bought. Represent the cost of five skirts.

85. A store advertises skirts for $x - 5$ dollars and allows an additional two-dollar reduction on each skirt if three or more skirts are purchased. Represent the cost of five skirts.

86. A store advertises skirts for $x - 5$ dollars and tops for $2x - 3$ dollars. Represent the cost of two skirts and three tops.

5-4 MULTIPLYING POLYNOMIALS

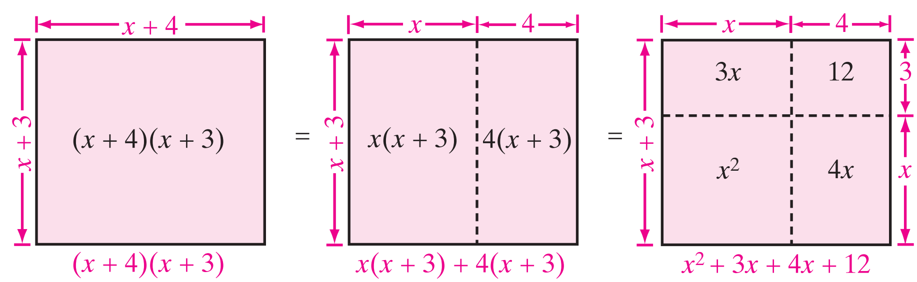
As discussed in Section 5-3, to find the product $(x + 4)(a)$, we use the distributive property of multiplication over addition:

$$(x + 4)(a) = x(a) + 4(a)$$

Now, let us use this property to find the product of two binomials, for example, $(x + 4)(x + 3)$.

$$\begin{aligned} (x + 4)\quad (a) \quad &= x \quad (a) \quad + 4 \quad (a) \\ (x + 4)(x + 3) &= x(x + 3) + 4(x + 3) \\ &= x^2 + 3x + 4x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

This result can also be illustrated geometrically.



In general, for all a, b, c, and d:

$$\begin{aligned} \boldsymbol{(a + b)(c + d)} &= \boldsymbol{a(c + d) + b(c + d)} \\ &= \boldsymbol{ac + ad + bc + bd} \end{aligned}$$

Notice that each term of the first polynomial multiplies each term of the second.

At the right is a convenient vertical arrangement of the preceding multiplication, similar to the arrangement used in arithmetic multiplication. *Note that multiplication is done from left to right.*

$$\begin{array}{rr} & x + 3 \\ & \underline{x + 4} \\ x(x + 3) \rightarrow & x^2 + 3x \\ 4(x + 3) \rightarrow & \underline{\quad + 4x + 12} \\ & x^2 + 7x + 12 \end{array}$$

The word **FOIL** serves as a convenient way to remember the steps necessary to multiply two binomials.

$$\begin{aligned} & \qquad\qquad\quad \textbf{F}\text{irst} \quad \textbf{O}\text{utside} \quad \textbf{I}\text{nside} \quad \textbf{L}\text{ast} \\ (2x - 5)(x + 4) &= 2x(x) + 2x(+4) - 5(x) - 5(+4) \\ &= 2x^2 + 8x - 5x - 20 \\ &= 2x^2 + 3x - 20 \end{aligned}$$

Procedure

To multiply a polynomial by a polynomial, first arrange each polynomial in descending or ascending powers of the same variable. Then use the distributive property: multiply each term of the first polynomial by each term of the other.

EXAMPLE 1

Simplify: $(3x - 4)(4x + 5)$

Solution

METHOD 1

$$\begin{aligned}(3x - 4)(4x + 5) &= 3x(4x + 5) - 4(x + 5)\\ &= 12x^2 + 15x - 16x - 20\\ &= 12x^2 - x - 20\end{aligned}$$

METHOD 2

$$\begin{array}{rrr} 3x & -4 & \\ 4x & +5 & \\ \hline 12x^2 & -16x & \\ & +15x & -20 \\ \hline 12x^2 & -x & -20 \end{array}$$

Answer $12x^2 - x - 20$

EXAMPLE 2

Simplify: $(x^2 + 3xy + 9y^2)(x - 3y)$

Solution

$$\begin{aligned}(x^2 + 3xy + 9y^2)(x - 3y) &= x^2(x - 3y) + 3xy(x - 3y) + 9y^2(x - 3y)\\ &= x^3 - 3x^2y + 3x^2y - 9xy^2 + 9xy^2 - 27y^3\\ &= x^3 + 0x^2y + 0xy^2 - 27y^3\end{aligned}$$

Answer $x^3 - 27y^3$

EXAMPLE 3

Simplify: $(2x - 5)^2 - (x - 3)$

Solution

$$\begin{aligned}(2x - 5)^2 - (x - 3) &= (2x - 5)(2x - 5) - (x - 3)\\ &= 2x(2x) + 2x(-5) - 5(2x) - 5(-5) + (-x) + (+3)\\ &= 4x^2 - 10x - 10x + 25 - x + 3\\ &= 4x^2 - 21x + 28\end{aligned}$$

Answer $4x^2 - 21x + 28$

EXERCISES

Writing About Mathematics

1. The product of two binomials in simplest form can have four terms, three terms, or two terms.

a. When does the product of two binomials have four terms?

b. When is the product of two binomials a trinomial?

c. When is the product of two binomials a binomial?

2. Burt wrote $(a + 3)^2$ as $a^2 + 9$. Prove to Burt that he is incorrect.

Developing Skills

In 3–35, write each product as a polynomial.

3. $(a + 2)(a + 3)$ **4.** $(x - 5)(x - 3)$ **5.** $(d + 9)(d - 3)$

6. $(x - 7)(x - 2)$ **7.** $(m + 3)(m - 7)$ **8.** $(t + 15)(t - 6)$

9. $(b - 8)(b - 10)$ **10.** $(6 + y)(5 + y)$ **11.** $(8 - e)(6 - e)$

12. $(12 - r)(6 + r)$ **13.** $(x + 5)(x - 5)$ **14.** $(2y + 7)(2y - 7)$

15. $(5a + 9)(5a - 9)$ **16.** $(2x + 1)(x - 6)$ **17.** $(5y - 2)(3y - 1)$

18. $(2x + 3)(2x - 3)$ **19.** $(3d + 8)(3d - 8)$ **20.** $(x + y)(x + y)$

21. $(a - b)(a + b)$ **22.** $(a + b)(a + b)$ **23.** $(a + b)^2$

24. $(x - 4y)(x + 4y)$ **25.** $(x - 4y)^2$ **26.** $(9x - 5y)(2x + 3y)$

27. $(r^2 + 5)(r^2 - 2)$ **28.** $(x^2 - y^2)(x^2 + y^2)$ **29.** $(x + 2)(x^2 + 3x + 5)$

30. $(2c + 1)(2c^2 - 3c + 1)$ **31.** $(3 - 2a - a^2)(5 - 2a)$ **32.** $(2x + 1)(3x - 4)(x + 3)$

33. $(x + 4)(x + 4)(x + 4)$ **34.** $(a + 5)^3$ **35.** $(x - y)^3$

In 36–43, simplify each expression.

36. $(x + 7)(x - 2) - x^2$ **37.** $2(3x + 1)(2x - 3) + 14x$

38. $r(r - 2) - (r - 5)$ **39.** $8x^2 - (4x + 3)(2x - 1)$

40. $(x + 4)(x + 3) - (x - 2)(x - 5)$ **41.** $(3y + 5)(2y - 3) - (y + 7)(5y - 1)$

42. $(y + 4)^2 - (y - 3)^2$ **43.** $a[(a + 2)(a - 2) - 4]$

Applying Skills

In 44–46, use grouping symbols to write an algebraic expression that represents the answer. Then, express each answer as a polynomial in simplest form.

44. The length of a rectangle is $2x - 5$ and its width is $x + 7$. Express the area of the rectangle as a trinomial.

45. The dimensions of a rectangle are represented by $11x - 8$ and $3x + 5$. Represent the area of the rectangle as a trinomial.

46. A train travels at a rate of $(15x + 100)$ kilometers per hours.

a. Represent the distance it can travel in $(x + 3)$ hours as a trinomial.

b. If $x = 2$, how fast does the train travel?

c. If $x = 2$, how far does it travel in $(x + 3)$ hours?

5-5 DIVIDING POWERS THAT HAVE THE SAME BASE

We know that any nonzero number divided by itself is 1. Therefore, $x \div x = 1$ and $y^3 \div y^3 = 1$.

In general, when $x \neq 0$ and a is a positive integer:

$$x^a \div x^a = 1$$

Therefore,

$$\frac{x^5}{x^3} = \frac{x^2 \cdot x^3}{x^3} = x^2 \cdot 1 = x^2$$

$$\frac{y^9}{y^4} = \frac{y^5 \cdot y^4}{y^4} = y^5 \cdot 1 = y^5$$

$$\frac{c^5}{c} = \frac{c^4 \cdot c}{c} = c^4 \cdot 1 = c^4$$

These same results can be obtained by using the relationship between division and multiplication:

If $a \cdot b = c$, then $c \div b = a$.

- Since $x^2 \cdot x^3 = x^5$, then $x^5 \div x^3 = x^2$.
- Since $y^5 \cdot y^4 = y^9$, then $y^9 \div y^4 = y^5$.
- Since $c^4 \cdot c = c^5$, then $c^5 \div c^1 = c^4$.

Observe that the exponent in each quotient is the difference between the exponent of the dividend and the exponent of the divisor.

In general, when $x \neq 0$ and a and b are positive integers with $a > b$:

$$x^a \div x^b = x^{a-b}$$

Procedure

To divide powers of the same base, find the exponent of the quotient by subtracting the exponent of the divisor from the exponent of the dividend. The base of the quotient is the same as the base of the dividend and of the divisor.

EXAMPLE 1

Simplify by performing each indicated division.

a. $x^9 \div x^5$ **b.** $y^5 \div y$ **c.** $c^5 \div c^5$ **d.** $10^5 \div 10^3$

Answers **a.** $x^{9-5} = x^4$ **b.** $y^{5-1} = y^4$ **c.** 1 **d.** $10^{5-3} = 10^2$

EXAMPLE 2

Write $\frac{5^7 \cdot 5^4}{5^8}$ in simplest form:

a. by using the rules for multiplying and dividing powers with like bases.

b. by using a calculator.

Solution **a.** First simplify the numerator. Then, apply the rule for division of powers with the same base.

$$\frac{5^7 \cdot 5^4}{5^8} = \frac{5^{7+4}}{5^8} = \frac{5^{11}}{5^8} = 5^{11-8} = 5^3 = 125$$

b. On a calculator:

ENTER: 5 [^] 7 [×] 5 [^] 4 [÷] 5 [^] 8 [ENTER]

DISPLAY:

```
5^7*5^4/5^8
                125
```

Answer 125

EXERCISES

Writing About Mathematics

1. Coretta said that $5^4 \div 5 = 1^4$. Do you agree with Coretta? Explain why or why not.

2. To evaluate the expression $\frac{3^8}{3^5 \cdot 3^2}$,

a. in what order should the operations be performed? Explain your answer.

b. does $\frac{3^8}{3^5 \cdot 3^2} = 3^8 \div 3^5 \cdot 3^2$? Explain why or why not.

Developing Skills

In 3–18, divide in each case.

3. $x^8 \div x^2$ **4.** $a^{10} \div a^5$ **5.** $c^5 \div c^4$ **6.** $x^7 \div x^7$

7. $\frac{e^9}{e^3}$ **8.** $\frac{m^{12}}{m^4}$ **9.** $\frac{n^{10}}{n^9}$ **10.** $\frac{r^6}{r^6}$

11. $x^8 \div x$ **12.** $z^{10} \div z$ **13.** $t^5 \div t$ **14.** $2^5 \div 2^2$

15. $10^6 \div 10^4$ **16.** $3^4 \div 3^2$ **17.** $5^3 \div 5$ **18.** $10^4 \div 10$

In 19–24, divide in each case. (All exponents are positive integers.)

19. $x^{5a} \div x^{2a}$ **20.** $y^{10b} \div y^{2b}$ **21.** $r^c \div r^d\ (c > d)$

22. $s^x \div s^2\ (x > 2)$ **23.** $a^b \div a^b$ **24.** $2^a \div 2^b\ (a > b)$

In 25–32: **a.** Simplify each expression by using the rules for multiplying and dividing powers with like bases. **b**. Evaluate the expression using a calculator. Compare your answers to parts **a** and **b**.

25. $\frac{2^3 \cdot 2^4}{2^2}$ **26.** $\frac{5^8}{5^4 \cdot 5}$ **27.** $\frac{10^2 \cdot 10^3}{10^4}$ **28.** $\frac{3^3 \cdot 3^2}{3^2}$

29. $\frac{10^6}{10^2 \cdot 10^4}$ **30.** $\frac{10^8 \cdot 10^2}{(10^5)^2}$ **31.** $\frac{6^4 \cdot 6^9}{6^2 \cdot 6^3}$ **32.** $\frac{4^5 \cdot 4^5}{(4^2)^4}$

In 33–35, tell whether each sentence is true or false.

33. $100^{99} \div 10^{98} = 100^2$ **34.** $a^6 \div a^2 = a^4\ (a \neq 0)$ **35.** $450^{45} \div 450^{40} = 1^5$

5-6 POWERS WITH ZERO AND NEGATIVE EXPONENTS

Integers that are negative or zero, such as 0, −1, and −2 can also be used as exponents. We will define powers having zero and negative integral exponents in such a way that the properties that were valid for positive integral exponents will also be valid for zero and negative integral exponents. In other words, the following properties will be true when the exponents a and b are positive integers, negative integers, or 0:

$$x^a \cdot x^b = x^{a+b} \qquad x^a \div x^b = x^{a-b} \qquad (x^a)^b = x^{ab}$$

The Zero Exponent

We know that, for $x \neq 0$, $\frac{x^3}{x^3} = 1$. If $\frac{x^3}{x^3} = x^{3-3} = x^0$ is to be a meaningful statement, we must let $x^0 = 1$ since x^0 and 1 are each equal to $\frac{x^3}{x^3}$. This leads to the following definition:

DEFINITION

$x^0 = 1$ if x is a number such that $x \neq 0$.

It can be shown that all the laws of exponents remain valid when x^0 is defined as 1. For example:

- Using the definition $10^0 = 1$, we have $10^3 \cdot 10^0 = 10^3 \cdot 1 = 10^3$

- Using the law of exponents, we have $10^3 \cdot 10^0 = 10^{3+0} = 10^3$.

The two procedures result in the same product.

- Using the definition $10^0 = 1$, we have $10^3 \div 10^0 = 10^3 \div 1 = 10^3$
- Using the law of exponents, we have $10^3 \div 10^0 = 10^{3-0} = 10^3$.

The two procedures result in the same quotient.

The definition $x^0 = 1$ $(x \neq 0)$ permits us to say that the zero power of any number except 0 equals 1.

$$4^0 = 1 \qquad (-4)^0 = 1 \qquad (4x)^0 = 1 \qquad (-4x)^0 = 1$$

A calculator will return this value. For example, to evaluate 4^0:

ENTER: 4 **^** 0 **ENTER**

DISPLAY:
```
4^0
                1
```

Note that $4x^0 = 4^1 \cdot x^0 = 4 \cdot 1 = 4$ but $(4x)^0 = 4^0 \cdot x^0 = 1 \cdot 1 = 1$.

The Negative Integral Exponent

We know that, for $x \neq 0$,

$$\frac{x^3}{x^5} = \frac{1 \cdot x^3}{x^2 \cdot x^3} = \frac{1}{x^2} \cdot \frac{x^3}{x^3} = \frac{1}{x^2} \cdot 1 = \frac{1}{x^2}.$$

If $\frac{x^3}{x^5} = x^{3-5} = x^{-2}$ is to be a meaningful statement, we must let $x^{-2} = \frac{1}{x^2}$ since x^{-2} and $\frac{1}{x^2}$ are each equal to $\frac{x^3}{x^5}$. This leads to the following definition:

DEFINITION

$x^{-n} = \frac{1}{x^n}$ if $x \neq 0$.

A graphing calculator will return equal values for x^{-2} and $\frac{1}{x^2}$. For example, let $x = 5$.

Evaluate 5^{-2}.

ENTER: 5 **^** **(-)** 2 **ENTER**

DISPLAY:
```
5^-2
              .04
```

Evaluate $\frac{1}{5^2}$.

ENTER: 1 **÷** 5 **^** 2 **ENTER**

DISPLAY:
```
1/5^2
              .04
```

It can be shown that all the laws of exponents remain valid if x^{-n} is defined as $\frac{1}{x^n}$. For example:

- Using the definition $2^{-4} = \frac{1}{2^4}$, we have $2^2 \cdot 2^{-4} = 2^2 \cdot \frac{1}{2^4} = \frac{2^2}{2^4} = \frac{1}{2^2} = 2^{-2}$.
- Using the law of exponents, we have $2^2 \cdot 2^{-4} = 2^{2+(-4)} = 2^{-2}$.

The two procedures give the same result.

Now we can say that, for all integral values of a and b,

$$\frac{x^a}{x^b} = x^{a-b} \; (x \neq 0)$$

EXAMPLE 1

Transform each given expression into an equivalent one with a positive exponent.

	Answers
a. 4^{-3}	$= \frac{1}{4^3}$
b. 10^{-1}	$= \frac{1}{10^1} = \frac{1}{10}$
c. $\frac{1}{2^{-5}}$	$= 1 \div 2^{-5} = 1 \div \frac{1}{2^5} = 1 \times \frac{2^5}{1} = 2^5$
d. $\left(\frac{5}{3}\right)^{-2}$	$= \frac{5^{-2}}{3^{-2}} = \frac{1}{5^2} \div \frac{1}{3^2} = \frac{1}{5^2} \times \frac{3^2}{1} = \frac{3^2}{5^2} = \left(\frac{3}{5}\right)^2$

EXAMPLE 2

Compute the value of each expression.

	Answers
a. 3^0	$= 1$
b. 10^{-2}	$= \frac{1}{10^2} = \frac{1}{100}$
c. $(-5)^0 + 2^{-4}$	$= 1 + \frac{1}{2^4} = 1 + \frac{1}{16} = 1\frac{1}{16}$
d. $6(3^{-3})$	$= 6\left(\frac{1}{3^3}\right) = 6\left(\frac{1}{27}\right) = \frac{6}{27} = \frac{2}{9}$

EXAMPLE 3

Use the laws of exponents to perform the indicated operations.

	Answers		*Answers*
a. $2^7 \cdot 2^{-3}$	$= 2^{7+(-3)} = 2^4$	**b.** $3^{-6} \div 3^{-2}$	$= 3^{-6-(-2)} = 3^{-6+2} = 3^{-4}$
c. $(x^4)^{-3}$	$= x^{4(-3)} = x^{-12}$	**d.** $(y^{-2})^{-4}$	$= y^{-2(-4)} = y^8$

EXERCISES

Writing About Mathematics

1. Sasha said that for all $x \neq 0$, x^{-2} is a positive number less than 1. Do you agree with Sasha? Explain why or why not.
2. Brandon said that, when n is a whole number, the number 10^n when written in ordinary decimal notation uses $n + 1$ digits. Do you agree with Brandon? Explain why or why not.

Developing Skills

In 3–7, transform each given expression into an equivalent expression involving a positive exponent.

3. 10^{-4} **4.** 2^{-1} **5.** $\left(\frac{2}{3}\right)^{-2}$ **6.** $m^{-6}, m \neq 0$ **7.** $r^{-3}, r \neq 0$

In 8–19, compute each value using the definitions of zero and negative exponents. Compare your answers with the results obtained using a calculator.

8. 10^0 **9.** $(-4)^0$ **10.** -4^0 **11.** 3^{-2}

12. 10^{-1} **13.** 10^{-2} **14.** 10^{-3} **15.** $4(10)^{-2}$

16. $1.5(10)^{-3}$ **17.** $7^0 + 6^{-2}$ **18.** $\left(\frac{1}{2}\right)^0 + 3^{-3}$ **19.** $-2 \cdot 4^{-1}$

In 20–27, use the laws of exponents to perform each indicated operation.

20. $10^{-2} \cdot 10^5$ **21.** $3^{-4} \cdot 3^{-2}$ **22.** $10^{-3} \div 10^{-5}$ **23.** $(4^{-2})^2 \div 4^{-4}$

24. $3^4 \div 3^0$ **25.** $(4^{-1})^2$ **26.** $(3^{-3})^{-2}$ **27.** $2^0 \cdot 2^{-5}$

28. Find the value of $7x^0 - (6x)^0, (x \neq 0)$.

29. Find the value of $5x^0 + 2x^{-1}$ when $x = 4$.

5-7 SCIENTIFIC NOTATION

Scientists and mathematicians often work with numbers that are very large or very small. In order to write and compute with such numbers more easily, these workers use **scientific notation**. A number is expressed in scientific notation when it is written as the product of two quantities: the first is a number greater than or equal to 1 but less than 10, and the second is a power of 10. In other words, a number is in scientific notation when it is written as

$$a \times 10^n$$

where $1 \leq a < 10$ and n is an integer.

Writing Numbers in Scientific Notation

To write a number in scientific notation, first write it as the product of a number between 1 and 10 times a power of 10. Then express the power of 10 in exponential form.

The table at the right shows some integral powers of 10. When the exponent is a positive integer, the power can be written as 1 followed by the number of 0's equal to the exponent of 10. When the exponent is a negative integer, the power can be written as a decimal value with the number of decimal places equal to the absolute value of the exponent of 10.

Powers of 10
$10^5 = 100,000$
$10^4 = 10,000$
$10^3 = 1,000$
$10^2 = 100$
$10^1 = 10$
$10^0 = 1$
$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$
$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$
$10^{-3} = \frac{1}{10^3} = \frac{1}{1,000} = 0.001$
$10^{-4} = \frac{1}{10^4} = \frac{1}{10,000} = 0.0001$

$$3,000,000 = 3 \times 1,000,000 = 3 \times 10^6$$
$$780 = 7.8 \times 100 = 7.8 \times 10^2$$
$$3 = 3 \times 1 = 3 \times 10^0$$
$$0.025 = 2.5 \times 0.01 = 2.5 \times 10^{-2}$$
$$0.0003 = 3 \times 0.0001 = 3 \times 10^{-4}$$

When writing a number in scientific notation, keep in mind the following:

- A number equal to or greater than 10 has a positive exponent of 10.
- A number equal to or greater than 1 but less than 10 has a zero exponent of 10.
- A number between 0 and 1 has a negative exponent of 10.

EXAMPLE 1

The distance from the earth to the sun is approximately 93,000,000 miles. Write this number in scientific notation.

Solution *How to Proceed*

(1) Write the number, placing a decimal point after the last digit. — 93,000,000.

(2) Place a caret (^) after the first nonzero digit so that replacing the caret with a decimal point will give a number between 1 and 10. — 9‸3,000,000.

(3) Count the number of digits between the caret and the decimal point. This is the exponent of 10 in scientific notation. The exponent is positive because the given number is greater than 10. — $9_\wedge\underbrace{3,000,000}_{7}.$

(4) Write the number in the form $a \times 10^n$, where where a is found by replacing the caret with a decimal point and n is the exponent found in Step 3. $\quad 9.3 \times 10^7$

Answer 9.3×10^7

EXAMPLE 2

Express 0.0000029 in scientific notation.

Solution Since the number is between 0 and 1, the exponent will be negative.

Place a caret after the first nonzero digit to indicate the position of the decimal point in scientific notation.

Answer $0.0000029 = 0.\underbrace{000002}_{6}{}_\wedge 9 = 2.9 \times 10^{-6}$

Graphing calculators can be placed in scientific notation mode and will return the results shown in Examples 1 and 2 when the given numbers are entered.

ENTER: **MODE** **▶** **ENTER** **CLEAR**

.0000029 **ENTER**

DISPLAY:

```
.0000029
           2.9E-6
```

This display is read as 2.9×10^{-6}, where the integer following "E" is the exponent to the base 10 used to write the number in scientific notation.

Changing to Ordinary Decimal Notation

We can change a number that is written in scientific notation to ordinary decimal notation by expanding the power of 10 and then multiplying the result by the number between 1 and 10.

EXAMPLE 3

The approximate population of the United States is 2.81×10^8. Find the approximate number of people in the United States.

Solution *How to Proceed*

(1) Evaluate the second factor, which is a power of 10: $2.81 \times 10^8 = 2.81 \times 100{,}000{,}000$

(2) Multiply the factors: $2.81 \times 10^8 = 281{,}000{,}000$

Answer 281,000,000 people

Note: We could have multiplied 2.81 by 10^8 quickly by moving the decimal point in 2.81 eight places to the right.

EXAMPLE 4

The diameter of a red blood corpuscle is expressed in scientific notation as 7.5×10^{-4} centimeters. Write the number of centimeters in the diameter as a decimal fraction.

Solution *How to Proceed*

(1) Evaluate the second factor, which is a power of 10: $7.5 \times 10^{-4} = 7.5 \times 0.0001$

(2) Multiply the factors: $7.5 \times 10^{-4} = 0.00075$

Answer 0.00075 cm

Note: We could have multiplied 7.5 by 10^{-4} quickly by moving the decimal point in 7.5 four places to the left.

EXAMPLE 5

Use a calculator to find the product: $45{,}000 \times 570{,}000$.

Calculator Solution ENTER: 45000 [×] 570000 [ENTER]

DISPLAY:

```
45000*570000
        2.565E10
```

A calculator will shift to scientific notation when the number is too large or too small for the display. The number in this display can be changed to decimal notation by using the procedure shown in Examples 3 and 4.

Answer $2.565 \times 10^{10} = 2.565 \times 10{,}000{,}000{,}000 = 25{,}650{,}000{,}000$

EXAMPLE 6

Use a calculator to find the mass of 2.70×10^{15} hydrogen atoms if the mass of one hydrogen atom is 1.67×10^{-24} grams. Round the answer to three significant digits.

Solution Multiply the mass of one hydrogen atom by the number of hydrogen atoms.

$$\begin{aligned} &(1.67 \times 10^{-24}) \times (2.70 \times 10^{15}) \\ &= (1.67 \times 2.70) \times (10^{-24} \times 10^{15}) \\ &= (1.67 \times 2.70) \times (10^{-24+15}) \\ &= 4.509 \times 10^{-9} \end{aligned}$$

Round 4.509 to 4.51, which has three significant digits.

Answer $4.51 \times 10^{-9} = 4.51 \times 0.000000001 = 0.00000000451$ grams

Calculator Solution Use a calculator to multiply the mass of one hydrogen atom by the number of hydrogen atoms. Enter the numbers in scientific notation.

ENTER: 1.67 **2nd** **EE** **(-)** 24 **×** 2.7 **2nd** **EE** 15 **ENTER**

DISPLAY:

```
1.67E-24*2.7E15
          4.509E-9
```

Round 4.509 to three significant digits.

Answer $4.51 \times 10^{-9} = 4.51 \times 0.000000001 = 0.00000000451$ grams

EXERCISES

Writing About Mathematics

1. Jared said that when a number is in scientific notation, $a \times 10^n$, the number of digits in a is the number of significant digits. Do you agree with Jared? Explain why or why not.
2. When Corey wanted to enter 2.54×10^{-5} into his calculator, he used this sequence of keys: 2.54 **×** **2nd** **EE** **−** 5 **ENTER**. Is this a correct way to enter the number? Explain why or why not.

Developing Skills

In 3–8, write each number as a power of 10.

3. 100 **4.** 10,000 **5.** 0.01

6. 0.0001 **7.** 1,000,000,000 **8.** 0.0000001

In 9–20, find the number that is expressed by each numeral.

9. 10^7 **10.** 10^{10} **11.** 10^{-3} **12.** 10^{-5}

13. 3×10^5 **14.** 4×10^8 **15.** 6×10^{-1} **16.** 9×10^{-7}

17. 1.3×10^4 **18.** 8.3×10^{-10} **19.** 1.27×10^3 **20.** 6.14×10^{-2}

In 21–32, find the value of n that will make each resulting statement true.

21. $120 = 1.2 \times 10^n$ **22.** $9{,}300 = 9.3 \times 10^n$ **23.** $5{,}280 = 5.28 \times 10^n$

24. $0.00161 = 1.61 \times 10^n$ **25.** $0.0000760 = 7.60 \times 10^n$ **26.** $52{,}000 = 5.2 \times 10^n$

27. $0.00000000375 = 3.75 \times 10^n$ **28.** $872{,}000{,}000 = 8.72 \times 10^n$ **29.** $0.800 = 8.00 \times 10^n$

30. $2.54 = 2.54 \times 10^n$ **31.** $0.00456 = 4.56 \times 10^n$ **32.** $7{,}123{,}000 = 7.123 \times 10^n$

In 33–44, express each number in scientific notation.

33. 8,400 **34.** 27,000 **35.** 54,000,000 **36.** 320,000,000

37. 0.00061 **38.** 0.0000039 **39.** 0.0000000140 **40.** 0.156

41. 453,000 **42.** 0.00381 **43.** 375,000,000 **44.** 0.0000763

In 45–48, compute the result of each operation. Using the correct number of significant digits: **a.** write the result in scientific notation, **b.** write the result in ordinary decimal notation.

45. $(2.9 \times 10^3)(3.0 \times 10^{-3})$ **46.** $(2.55 \times 10^{-2})(3.00 \times 10^{-3})$

47. $(7.50 \times 10^4) \div (2.5 \times 10^3)$ **48.** $(6.80 \times 10^{-5}) \div (3.40 \times 10^{-8})$

Applying Skills

In 49–52, express each number in scientific notation.

49. A light-year, which is the distance light travels in 1 year, is approximately 9,500,000,000,000 kilometers.

50. A star that is about 12,000,000,000,000,000,000,000 miles away can be seen by the Palomar telescope.

51. The radius of an electron is about 0.0000000000005 centimeters.

52. The diameter of some white blood corpuscles is approximately 0.0008 inches.

In 53–57, express each number in ordinary decimal notation.

53. The diameter of the universe is 2×10^9 light-years.

54. The distance from the earth to the moon is 2.4×10^5 miles.

55. In a motion-picture film, the image of each picture remains on the screen approximately 6×10^{-2} seconds.

56. Light takes about 2×10^{-8} seconds to cross a room.

57. The mass of the earth is approximately 5.9×10^{24} kilograms.

5-8 DIVIDING BY A MONOMIAL

Dividing a Monomial by a Monomial

We know that

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

We can rewrite this equality interchanging the left and right members.

$$\frac{ac}{bd} = \frac{a}{b} \cdot \frac{c}{d}$$

Using this relationship, we can write:

$$\frac{-30x^6}{2x^4} = \frac{-30}{2} \cdot \frac{x^6}{x^4} = -15x^2$$

$$\frac{-21a^5b^4}{-3a^4b} = \frac{-21}{-3} \cdot \frac{a^5}{a^4} \cdot \frac{b^4}{b} = 7a^1b^3 = 7ab^3$$

$$\frac{12y^2z^2}{4y^2z} = \frac{12}{4} \cdot \frac{y^2}{y^2} \cdot \frac{z^2}{z} = 3y^0z^1 = 3 \cdot 1 \cdot z = 3z$$

Procedure

To divide a monomial by a monomial:

1. Divide the numerical coefficients.
2. When variable factors are powers of the same base, divide by subtracting exponents.
3. Multiply the quotients from steps 1 and 2.

If the area of a rectangle is 42 and its length is 6, we can find its width by dividing the area, 42, by the length, 6. Thus, $42 \div 6 = 7$, which is the width.

Similarly, if the area of a rectangle is represented by $42x^2$ and its length by $6x$, we can find its width by dividing the area, $42x^2$, by the length, $6x$:

$$42x^2 \div 6x = 7x$$

Therefore, the width can be represented by $7x$.

EXAMPLE 1

Divide: *Answers*

a. $\frac{24a^5}{-3a^2} = \frac{24}{-3} \cdot \frac{a^5}{a^2} = -8a^3$

b. $\frac{-18x^3y^2}{-6x^2y} = \frac{-18}{-6} \cdot \frac{x^3}{x^2} \cdot \frac{y^2}{y} = 3xy$

c. $\frac{20a^3c^4d^2}{-5a^3c^3} = \frac{20}{-5} \cdot \frac{a^3}{a^3} \cdot \frac{c^4}{c^3} \cdot d^2 = -4(1)cd^2 = -4cd^2$

EXAMPLE 2

The area of a rectangle is $24x^4y^3$. Express, in terms of x and y, the length of the rectangle if the width is $3xy^2$.

Solution The length of a rectangle can be found by dividing the area by the width.

$$\frac{24x^4y^3}{3xy^2} = 8x^3y \text{ Answer}$$

Dividing a Polynomial by a Monomial

We know that to divide by a number is the same as to multiply by its reciprocal. Therefore,

$$\frac{a+c}{b} = \frac{1}{b}(a+c) = \frac{a}{b} + \frac{c}{b}$$

Similarly,

$$\frac{2x+2y}{2} = \frac{1}{2}(2x+2y) = \frac{2x}{2} + \frac{2y}{2} = x + y$$

and

$$\frac{21a^2b - 3ab}{3ab} = \frac{1}{3ab}(21a^2b - 3ab) = \frac{21a^2b}{3ab} - \frac{3ab}{3ab} = 7a - 1$$

Usually, the two middle steps are done mentally.

Procedure

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

EXAMPLE 3

Divide: *Answers*

a. $(8a^5 - 6a^4) \div 2a^2$ $= 4a^3 - 3a^2$

b. $\dfrac{24x^3y^4 - 18x^2y^2 - 6xy}{-6xy}$ $= -4x^2y^3 + 3xy + 1$

EXERCISES

Writing About Mathematics

1. Mikhail divided $(12ab^2 + 6ab)$ by $(6ab)$ and got $2b$ for his answer. Explain to Mikhail why his answer is incorrect.

2. Angelique divided $(15cd + 11c)$ by $5c$ and got $(3d + 2.2)$ as her answer. Do you agree with Angelique? Explain why or why not.

Developing Skills

In 3–26, divide in each case.

3. $14x^2y^2 \div -7$

4. $-36y^{10} \div 6y^2$

5. $\dfrac{18x^6}{2x^2}$

6. $\dfrac{5x^2y^3}{-5y^3}$

7. $\dfrac{-49c^4b^3}{7c^2b^2}$

8. $\dfrac{-24x^2y}{-3xy}$

9. $\dfrac{-56abc}{8abc}$

10. $\dfrac{-27xyz}{9xz}$

11. $(14x + 7) \div 7$

12. $\dfrac{cm + cn}{c}$

13. $\dfrac{tr - r}{r}$

14. $\dfrac{8c^2 - 12d^2}{-4}$

15. $\dfrac{p + prt}{p}$

16. $\dfrac{y^2 - 5y}{-y}$

17. $\dfrac{18d^3 + 12d^2}{6d}$

18. $\dfrac{18r^5 + 12r^3}{6r^2}$

19. $\dfrac{9y^9 - 6y^6}{-3y^3}$

20. $\dfrac{8a^3 - 4a^2}{-4a^2}$

21. $\dfrac{3ab^2 - 4a^2b}{ab}$

22. $\dfrac{4c^2d - 12cd^2}{4cd}$

23. $\dfrac{-2a^2 - 3a + 1}{-1}$

24. $\dfrac{2.4y^5 + 1.2y^4 - 0.6y^3}{-0.6y^3}$

25. $\dfrac{a^3 - 2a^2}{0.5a^2}$

26. $\dfrac{1.6cd - 4.0c^2d}{0.8cd}$

Applying Skills

27. If five oranges cost $15y$ cents, represent the average cost of one orange.

28. If the area of a triangle is $32ab$ and the base is $8a$, represent the height of the triangle.

29. If a train traveled $54r$ miles in 9 hours, represent the average distance traveled in 1 hour.

30. If $40ab$ chairs are arranged in $5a$ rows with equal numbers of chairs in each row, represent the number of chairs in one row.

5-9 DIVIDING BY A BINOMIAL

When we divide 736 by 32, we use repeated subtraction of multiples of 32 to determine how many times 32 is contained in 736. To divide a polynomial by a binomial, we will use a similar procedure to divide $x^2 + 6x + 8$ by $x + 2$.

How to Proceed

(1) Write the usual division form:

$$x + 2\overline{)x^2 + 6x + 8}$$

(2) Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient:

$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 6x + 8} \end{array}$$

(3) Multiply the whole divisor by the first term of the quotient. Write each term of the product under the like term of the dividend:

$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 6x + 8} \\ \underline{x^2 + 2x} \end{array}$$

(4) Subtract and bring down the next term of the dividend to obtain a new dividend:

$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 6x + 8} \\ \underline{x^2 + 2x } \\ 4x + 8 \end{array}$$

(5) Divide the first term of the new dividend by the first term of the divisor to obtain the next term of the quotient:

$$\begin{array}{r} x + 4 \\ x + 2\overline{)x^2 + 6x + 8} \\ \underline{x^2 + 2x } \\ 4x + 8 \end{array}$$

(6) Repeat steps (3) and (4), multiplying the whole divisor by the new term of the quotient. Subtract this product from the new dividend. Here the remainder is zero and the division is complete:

$$\begin{array}{r} x + 4 \\ x + 2\overline{)x^2 + 6x + 8} \\ \underline{x^2 + 2x } \\ 4x + 8 \\ \underline{4x + 8} \\ 0 \end{array}$$

The division can be checked by multiplying the quotient by the divisor to obtain the dividend:

$$\begin{aligned} (x + 4)(x + 2) &= x(x + 2) + 4(x + 2) \\ &= x^2 + 2x + 4x + 8 = x^2 + 6x + 8 \end{aligned}$$

EXAMPLE 1

Divide $5s + 6s^2 - 6$ by $2s + 3$ and check.

Solution First arrange the terms of the dividend in descending order: $6s^2 + 5s - 6$

$$\begin{array}{r}
3s - 2 \\
2s + 3\overline{)6s^2 + 5s - 6} \\
\underset{(-)}{6s^2 + 9s} \\
\hline
-4s - 6 \\
\underline{-4s - 6} \\
0
\end{array}$$

Check

$(3s - 2)(2s + 3)$

$= 3s(2s + 3) - 2(2s + 3)$

$= 6s^2 + 9s - 4s - 6$

$= 6s^2 + 5s - 6$ ✔

Note that we subtracted $9s$ from $5s$ by adding $-9s$ to $5s$.

Answer $3s - 2$

EXERCISES

Writing about Mathematics

1. Nate said that $\frac{x^3 - 1}{x + 1} = \frac{x^3}{x} + \frac{-1}{1} = x^2 - 1$. Is Nate correct? Explain why or why not.

2. Mason wrote $x^3 - 1$ as $x^3 + 0x^2 + 0x - 1$ before dividing by $x + 1$.
   a. Does $x^3 - 1 = x^3 + 0x^2 + 0x - 1$?
   b. Divide $x^3 - 1$ by $x - 1$ by writing $x^3 + 0x^2 + 0x - 1$ as the dividend. Check your answer to show that your computation is correct.

Developing Skills

In 3–14, divide and check.

3. $(b^2 + 5b + 6) \div (b + 3)$ **4.** $(y^2 + 3y + 2) \div (y + 2)$ **5.** $(m^2 - 8m + 7) \div (m - 1)$

6. $\frac{w^2 + 2w - 15}{w + 5}$ **7.** $\frac{y^2 + 21y + 68}{y + 17}$ **8.** $\frac{x^2 + 7x + 10}{x + 5}$

9. $(3a^2 - 8a + 4) \div (3a - 2)$ **10.** $(15t^2 - 19t - 56) \div (5t + 7)$ **11.** $\frac{10y^2 - y - 24}{2y + 3}$

12. $\frac{8 - 22c + 12c^2}{4c - 2}$ **13.** $(17x + 66 + x^2) \div (x + 6)$ **14.** $\frac{x^2 - 64}{x - 8}$

15. One factor of $x^2 - 4x - 21$ is $x - 7$. Find the other factor.

Applying Skills

16. The area of a rectangle is represented by $x^2 - 8x - 9$. If its length is represented by $x + 1$, how can the width be represented?

17. The area of a rectangle is represented by $3y^2 + 8y + 4$. If its length is represented by $3y + 2$, how can the width be represented?

CHAPTER SUMMARY

Two or more terms that contain the same variable, with corresponding variables having the same exponents, are called **like terms**. The sum of like terms is the sum of the coefficients of the terms times the common variable factor of the terms.

A term that has no variable in the denominator is called a **monomial**. A **polynomial** is the sum of monomials.

To subtract one polynomial from another, add the opposite of the polynomial to be subtracted (the subtrahend) to the polynomial from which it is to be subtracted (the minuend).

When x is a nonzero real number and a and b are integers:

$$x^a \cdot x^b = x^{a+b} \qquad (x^a)^b = x^{ab} \qquad x^a \div x^b = x^{a-b} \qquad x^0 = 1 \qquad x^{-a} = \frac{1}{x^a}$$

A number is in **scientific notation** when it is written as $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

If $x = a \times 10^n$. Then:

- When $x \geq 10$, n is positive.
- When $1 \leq x < 10$, n is zero.
- When $0 < x < 1$, n is negative.

To multiply a polynomial by a polynomial, multiply each term of one polynomial by each term of the other polynomial and write the product as the sum of these results in simplest form.

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial and write the quotient as the sum of these results.

To divide a polynomial by a binomial, subtract multiples of the divisor from the dividend until the remainder is 0 or of degree less than the degree of the divisor.

VOCABULARY

5-1 Term • Like terms (similar terms) • Unlike terms • Monomial • Polynomial • Binomial • Trinomial • Simplest form • Descending order • Ascending order

5-4 FOIL

5-5 Zero exponent ($x^0 = 1$) • Negative integral exponent ($x^{-n} = \frac{1}{x^n}$)

5-7 Scientific notation

REVIEW EXERCISES

1. Explain why scientific notation is useful.

2. Is it possible to write a general rule for simplifying an expression such as $a^n + b^n$?

In 3–17, simplify each expression.

3. $5bc - bc$ **4.** $3y^2 - 2y + y^2 - 8y - 2$ **5.** $5t - (4 - 8t)$

6. $8mg(-3g)$ **7.** $3x^2(4x^2 + 2x - 1)$ **8.** $(4x + 3)(2x - 1)$

9. $(-6ab^3)^2$ **10.** $(-6a + b)^2$ **11.** $(2a + 5)(2a - 5)$

12. $(2a - 5)^2$ **13.** $2x - x(2x - 5)$ **14.** $\frac{40b^3c^6}{-8b^2c}$

15. $5y + \frac{6y^4}{-2y^3}$ **16.** $\frac{6w^3 - 8w^2 + 2w}{2w}$ **17.** $\frac{x^2 + x - 30}{x - 5}$

In 18–21, use the laws of exponents to perform the operations, and simplify.

18. $3^5 \cdot 3^4$ **19.** $(7^3)^2$ **20.** $[2(10^2)]^3$ **21.** $12^0 + 12^{-2} \cdot 12$

In 22–25, express each number in scientific notation.

22. 5,800 **23.** 14,200,000 **24.** 0.00006 **25.** 0.00000277

In 26–29, find the decimal number that is expressed by each given numeral.

26. 4×10^4 **27.** 3.06×10^{-3} **28.** 9.7×10^8 **29.** 1.03×10^{-4}

30. Express the area of each of the gardens and the total area of the two gardens described in the chapter opener on page 167.

31. If the length of one side of a square is $2h + 3$, express in terms of h:

a. the perimeter of the square.

b. the area of the square.

32. The perimeter of a triangle is $41px$. If the lengths of two sides are $18px$ and $7px$, represent the length of the third side.

33. If the length of a rectangle can be represented by $x + 5$, and the area of the rectangle by $x^2 + 7x + 10$, find the polynomial that represents:

a. the width of the rectangle.

b. the perimeter of the rectangle.

34. The cost of a pizza is 20 cents less than 9 times the cost of a soft drink. If x represents the cost, in cents, of a soft drink, express in simplest form the cost of two pizzas and six soft drinks.

Exploration

Study the squares of two-digit numbers that end in 5. From what you observe, can you devise a method for finding the square of such a number mentally? Can this method be applied to the square of a three-digit number that ends in 5?

Study the squares of the integers from 1 to 12. From what you observe, can you devise a method that uses the square of an integer to find the square of the next larger integer?

CUMULATIVE REVIEW — CHAPTERS 1–5

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Which of the numbers listed below has the largest value?

   (1) $1\frac{2}{3}$ (2) 1.67 (3) $1.6\overline{7}$ (4) $\frac{12}{7}$

2. For which of the following values of x is $x^2 > x > \frac{1}{x}$?

   (1) 1 (2) 0 (3) 3 (4) $\frac{2}{3}$

3. Which of the numbers given below is not a rational number?

   (1) $\sqrt{2}$ (2) $1\frac{1}{2}$ (3) $1.\overline{3}$ (4) $\frac{7}{3}$

4. Which of the following inequalities is false?

   (1) $1.5 < 1\frac{1}{2}$ (2) $1.5 \leq 1\frac{1}{2}$ (3) $-1.5 < 1.5$ (4) $-1.5 < -1$

5. Which of the following identities is an illustration of the associative property?

   (1) $x + 7 = 7 + x$
   (2) $3(x + 7) = 3x + 3(7)$
   (3) $(x + 7) + 3 = 3 + (7 + x)$
   (4) $(x + 7) + 3 = x + (7 + 3)$

6. The formula $C = \frac{5}{9}(F - 32)$ can be used to find the Celsius temperature, C, for a given Fahrenheit temperature, F. What Celsius temperature is equal to a Fahrenheit temperature of 68°?

   (1) 3° (2) 20° (3) 35° (4) 180°

7. If the universe is the set of whole numbers, the solution set of $x \leq 3$ is

   (1) {0, 1, 2} (2) {0, 1, 2, 3} (3) {1, 2} (4) {1, 2, 3}

8. The perimeter of a square whose area is 81 square centimeters is

(1) 9 cm (2) 18 cm (3) 20.25 cm (4) 36 cm

9. In simplest form, $(2x - 4)^2 + 3(x + 1)$ is equal to

(1) $4x^2 - 13x - 13$
(2) $4x^2 - 13x + 19$
(3) $4x^2 + 3x + 19$
(4) $4x^2 + 3x - 13$

10. To the nearest tenth of a meter, the circumference of a circle whose radius is 12.0 meters is

(1) 37.6 m (2) 37.7 m (3) 75.3 m (4) 75.4 m

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

11. The formula for the volume V of a cone is $V = \frac{1}{3}Bh$ where B is the area of the base and h is the height. Solve the formula for h in terms of V and B.

12. Each of the numbers given below is different from the others. Explain in what way each is different.

2 7 77 84

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

13. Solve the given equation for x. Show each step of the solution and name the property that is used in each step.

$$3(x - 4) = 5x + 8$$

14. Simplify the following expression. Show each step of the simplification and name the property that you used in each step.

$$4a - 7 + (7 - 3a)$$

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

15. A small park is in the shape of a rectangle that measures 525 feet by 468 feet.

a. Find the number of feet of fencing that would be needed to enclose the park. Express your answer to the nearest foot.

b. If the entire park is to be planted with grass seed, find the number of square feet to be seeded. Express your answer to the correct number of significant digits based on the given dimensions.

c. The grass seed to be purchased is packaged in sacks, each of which holds enough seed to cover 25,000 square feet of ground. How many sacks of seed are needed to seed the park?

16. An ice cream stand sells single-dip cones for $1.75 and double-dip cones for $2.25. Yesterday, 500 cones were sold for $930. How many single-dip and how many double-dip cones were sold?