

CHAPTER

4

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FIRST DEGREE EQUATIONS AND INEQUALITIES IN ONE VARIABLE

An equation is an important problem-solving tool. A successful business person must make many decisions about business practices. Some of these decisions involve known facts, but others require the use of information obtained from equations based on expected trends.

For example, an equation can be used to represent the following situation. Helga sews hand-made quilts for sale at a local craft shop. She knows that the materials for the last quilt that she made cost \$76 and that it required 44 hours of work to complete the quilt. If Helga received \$450 for the quilt, how much did she earn for each hour of work, taking into account the cost of the materials?

Most of the problem-solving equations for business are complex. Before you can cope with complex equations, you must learn the basic principles involved in solving any equation.

4-1 SOLVING EQUATIONS USING MORE THAN ONE OPERATION

Some Terms and Definitions

An **equation** is a sentence that states that two algebraic expressions are equal. For example, $x + 3 = 9$ is an equation in which $x + 3$ is called the **left side**, or **left member**, and 9 is the **right side**, or **right member**.

An equation may be a *true* sentence such as $5 + 2 = 7$, a *false* sentence such as $6 - 3 = 4$, or an *open* sentence such as $x + 3 = 9$. The number that can replace the variable in an open sentence to make the sentence true is called a **root**, or a **solution**, of the equation. For example, 6 is a root of $x + 3 = 9$.

As discussed in Chapter 3, the replacement set or domain is the set of possible values that can be used in place of the variable in an open sentence. If no replacement set is given, the replacement set is the set of real numbers. The set consisting of all elements of the replacement set that are solutions of the open sentence is called the **solution set** of the open sentence. For example, if the replacement set is the set of real numbers, the solution set of $x + 3 = 9$ is $\{6\}$. If no element of the replacement set makes the open sentence true, the solution set is the empty or null set, \emptyset or $\{\}$. If every element of the domain satisfies an equation, the equation is called an **identity**. Thus, $5 + x = x - (-5)$ is an identity when the domain is the set of real numbers because every element of the domain makes the sentence true.

Two equations that have the same solution set are **equivalent equations**. To **solve an equation** is to find its solution set. This is usually done by writing simpler equivalent equations.

If not every element of the domain makes the sentence true, the equation is called a **conditional equation**, or simply an equation. Therefore, $x + 3 = 9$ is a conditional equation.

Properties of Equality

When two numerical or algebraic expressions are equal, it is reasonable to assume that if we change each in the same way, the resulting expressions will be equal. For example:

$$\begin{aligned} 5 + 7 &= 12 \\ (5 + 7) + 3 &= 12 + 3 \\ (5 + 7) - 8 &= 12 - 8 \\ -2(5 + 7) &= -2(12) \\ \frac{5 + 7}{3} &= \frac{12}{3} \end{aligned}$$

These examples suggest the following properties of equality:

Properties of Equality

- 1. The addition property of equality.** If equals are added to equals, the sums are equal.
- 2. The subtraction property of equality.** If equals are subtracted from equals, the differences are equal.
- 3. The multiplication property of equality.** If equals are multiplied by equals, the products are equal.
- 4. The division property of equality.** If equals are divided by nonzero equals, the quotients are equal.
- 5. The substitution principle.** In a statement of equality, a quantity may be substituted for its equal.

To solve an equation, you need to work backward or “undo” what has been done by using inverse operations. To undo the addition of a number, add its opposite. For example, to solve the equation $x + 7 = 19$, use the addition property of equality. Add the opposite of 7 to both sides.

$$\begin{array}{r} x + 7 = 19 \\ \underline{-7 \quad -7} \\ x = 12 \end{array}$$

The variable x is now alone on one side and it is easy to read the solution, $x = 12$.

To solve an equation in which the variable has been multiplied by a number, either divide by that number or multiply by its reciprocal. (Remember multiplying by the reciprocal is the same as dividing by the number.) To solve $6x = 24$, divide both sides by 6 or multiply both sides by $\frac{1}{6}$.

$$\begin{array}{r} 6x = 24 \\ \underline{6x = 24} \\ x = 4 \end{array} \quad \text{or} \quad \begin{array}{r} 6x = 24 \\ \underline{\frac{1}{6}(6x) = \frac{1}{6}(24)} \\ x = 4 \end{array}$$

To solve $\frac{x}{3} = 5$, multiply each side by the reciprocal of $\frac{1}{3}$ which is 3.

$$\begin{array}{r} \frac{x}{3} = 5 \\ (3)\frac{x}{3} = (3)5 \\ x = 15 \end{array}$$

In the equation $2x + 3 = 15$, there are two operations in the left side: multiplication and addition. In forming the left side of the equation, x was first multiplied by 2, and then 3 was added to the product. To solve this equation, we must undo these operations by using the inverse elements in the reverse order. Since the last operation was to add 3, the first step in solving the equation is to add its opposite, -3 , to both sides of the equation or subtract 3 from both sides

of the equation. Here we are using either the addition or the subtraction property of equality.

$$\begin{array}{r}
 2x + 3 = 15 \\
 2x + 3 + (-3) = 15 + (-3) \\
 2x = 12
 \end{array}
 \quad \text{or} \quad
 \begin{array}{r}
 2x + 3 = 15 \\
 \frac{-3}{2x} = \frac{-3}{12}
 \end{array}$$

Now we have a simpler equation that has the same solution set as the original and includes only multiplication by 2. To solve this simpler equation, we multiply both sides of the equation by $\frac{1}{2}$, the reciprocal of 2, or divide both sides of the equation by 2. Here we can use either the multiplication or the division property of equality.

$$\begin{array}{r}
 2x = 12 \\
 \frac{1}{2}(2x) = \frac{1}{2}(12) \\
 x = 6
 \end{array}
 \quad \text{or} \quad
 \begin{array}{r}
 2x = 12 \\
 \frac{2x}{2} = \frac{12}{2} \\
 x = 6
 \end{array}$$

After an equation has been solved, we **check** the equation, that is, we verify that the solution does in fact make the given equation true by replacing the variable with the solution and performing any computations.

$$\begin{array}{l}
 \text{Check: } 2x + 3 = 15 \\
 2(6) + 3 = 15 \\
 12 + 3 = 15 \\
 15 = 15 \checkmark
 \end{array}$$

To find the solution of the equation $2x + 3 = 15$, we used several properties of the four basic operations and of equality. The solution below shows the mathematical principle that we used in each step.

$2x + 3 = 15$	Given
$(2x + 3) + (-3) = 15 + (-3)$	Addition property of equality
$2x + [3 + (-3)] = 15 + (-3)$	Associative property of addition
$2x + 0 = 12$	Additive inverse property
$2x = 12$	Additive identity property
$\frac{1}{2}(2x) = \frac{1}{2}(12)$	Multiplication property of equality
$[\frac{1}{2}(2)]x = \frac{1}{2}(12)$	Associative property of multiplication
$1x = 6$	Multiplicative inverse property
$x = 6$	Multiplicative identity property

These steps and properties are necessary to justify the solution of an equation of this form. However, when solving an equation, we do not need to write each of the steps, as shown in the examples that follow.

EXAMPLE 1Solve and check: $7x + 15 = 71$ **Solution***How to Proceed*

- | | |
|--|--|
| (1) Write the equation: | $7x + 15 = 71$ |
| (2) Add -15 , the opposite of $+15$ to each side: | $\frac{-15}{7x} = \frac{-15}{56}$ |
| (3) Since multiplication and division are inverse operations, divide each side by 7: | $\frac{7x}{7} = \frac{56}{7}$
$x = 8$ |
| (4) Check the solution. Write the solution in place of x and perform the computations: | $\begin{aligned} 7x + 15 &= 71 \\ 7(8) + 15 &\stackrel{?}{=} 71 \\ 56 + 15 &\stackrel{?}{=} 71 \\ 71 &= 71 \checkmark \end{aligned}$ |

Answer $x = 8$ **Note:** The check is based on the substitution principle.**EXAMPLE 2**Find the solution set and check: $\frac{3}{5}x - 6 = -18$ **Solution**

$$\begin{aligned} \frac{3}{5}x - 6 &= -18 \\ \frac{3}{5}x - 6 + 6 &= -18 + 6 && \text{Addition property of equality} \\ \frac{3}{5}x &= -12 \\ \frac{5}{3}\left(\frac{3}{5}x\right) &= \frac{5}{3}(-12) && \text{Multiplication property of equality} \\ x &= -20 \end{aligned}$$

Check

$$\begin{aligned} \frac{3}{5}x - 6 &= -18 \\ \frac{3}{5}(-20) - 6 &\stackrel{?}{=} -18 \\ -12 - 6 &\stackrel{?}{=} -18 \\ -18 &= -18 \checkmark \end{aligned}$$

Answer The solution set is $\{-20\}$.**EXAMPLE 3**Solve and check: $7 - x = 9$ **Solution** **METHOD 1.** Think of $7 - x$ as $7 + (-1x)$.

$$\begin{array}{r} 7 + (-x) = 9 \\ -7 \quad -7 \\ \hline -x = 2 \end{array}$$

Addition property of equality

$$\begin{array}{r} \frac{-1x}{-1} = \frac{2}{-1} \\ x = -2 \end{array}$$

Division property of equality

Check

$$7 - x = 9$$

$$7 - (-2) \stackrel{?}{=} 9$$

$$7 + 2 \stackrel{?}{=} 9$$

$$9 = 9 \checkmark$$

METHOD 2. Add x to both sides of the equation so that the variable has a positive coefficient.

How to Proceed

(1) Write the equation: $7 - x = 9$

(2) Add x to each side of the equation: $7 - x + x = 9 + x$

$$7 = 9 + x$$

(3) Add -9 to each side of the equation: $-9 + 7 = -9 + 9 + x$

$$-2 = x$$

The check is the same as for Method 1.

Answer $\{-2\}$ or $x = -2$

EXERCISES

Writing About Mathematics

- Is it possible for the equation $2x + 5 = 0$ to have a solution in the set of positive real numbers? Explain your answer.
- Max wants to solve the equation $7x + 15 = 71$. He begins by multiplying both sides of the equation by $\frac{1}{7}$, the reciprocal of the coefficient of x .
 - Is it possible for Max to solve the equation if he begins in this way? If so, what would be the result of multiplying by $\frac{1}{7}$ and what would be his next step?
 - In this section you learned to solve the equation $7x + 15 = 71$ by first adding the opposite of 15, -15 , to both sides of the equation. Which method do you think is better? Explain your answer.

Developing Skills

In 3 and 4, write a complete solution for each equation, listing the property used in each step.

3. $3x + 5 = 35$

4. $\frac{1}{2}x - 1 = 15$

In 5–32, solve and check each equation.

- | | | | |
|-----------------------------|---------------------------|-----------------------------------|--|
| 5. $55 = 6a + 7$ | 6. $17 = 8c - 7$ | 7. $9 - 1x = 7$ | 8. $11 = 15t + 16$ |
| 9. $15 - a = 3$ | 10. $11 = -6d - 1$ | 11. $8 - y = 1$ | 12. $\frac{3a}{8} = 12$ |
| 13. $\frac{2}{3}x = -8$ | 14. $12 = \frac{3}{4}y$ | 15. $\frac{5t}{4} = \frac{45}{2}$ | 16. $-\frac{3}{5}m = 30$ |
| 17. $7.2 = \frac{4m}{5}$ | 18. $\frac{a}{4} + 9 = 5$ | 19. $-2 = \frac{y}{5} + 3$ | 20. $9d - \frac{1}{2} = 17\frac{1}{2}$ |
| 21. $4a + 0.2 = 5$ | 22. $4 = 3t - 0.2$ | 23. $\frac{1}{4}x + 11 = 5$ | 24. $13 = 5 - \frac{2}{3}y$ |
| 25. $\frac{4}{5}t + 7 = 47$ | 26. $0.04c + 1.6 = 0$ | 27. $15x + 14 = 19$ | 28. $8 = 18c - 1$ |
| 29. $\frac{1}{7} = 14 - x$ | 30. $0.8r + 19 = 20$ | 31. $\frac{1}{3}w + 6 = -2$ | 32. $842 - 162m = -616$ |

Applying Skills

33. The formula $F = \frac{9}{5}C + 32$ gives the relationship between the Fahrenheit temperature F and the Celsius temperature C . Solve the equation $59 = \frac{9}{5}C + 32$ to find the temperature in degrees Celsius when the Fahrenheit temperature is 59° .
34. When Kurt orders from a catalog, he pays \$3.50 for shipping and handling in addition to the cost of the goods that he purchases. Kurt paid \$33.20 when he ordered six pairs of socks. Solve the equation $6x + 3.50 = 33.20$ to find x , the price of one pair of socks.
35. When Mattie rents a car for one day, the cost is \$29.00 plus \$0.20 a mile. On her last trip, Mattie paid \$66.40 for the car for one day. Find the number of miles, m , that Mattie drove by solving the equation $29 + 0.20x = 66.40$.
36. On his last trip to the post office, Hal paid \$4.30 to mail a package and bought some 39-cent stamps. He paid a total of \$13.66. Find s , the number of stamps that he bought, by solving the equation $0.39s + 4.30 = 13.66$.

4-2 SIMPLIFYING EACH SIDE OF AN EQUATION

An equation is often written in such a way that one or both sides are not in simplest form. Before starting to solve the equation by using additive and multiplicative inverses, you should simplify each side by removing parentheses if necessary and adding like terms.

Recall that an algebraic expression that is a number, a variable, or a product or quotient of numbers and variables is called a *term*. **First-degree equations in one variable** contain two kinds of terms, terms that are constants and terms that contain the variable to the first power only.

Like and Unlike Terms

Two or more terms that contain the same variable or variables, with corresponding variables having the same exponents, are called **like terms** or **similar terms**. For example, the following pairs are like terms.

$$6k \text{ and } k \quad 5x^2 \text{ and } -7x^2 \quad 9ab \text{ and } 0.4ab \quad \frac{9}{2}x^2y^3 \text{ and } -\frac{11}{3}x^2y^3$$

Two terms are **unlike terms** when they contain different variables, or the same variable or variables with different exponents. For example, the following pairs are unlike terms.

$$3x \text{ and } 4y \quad 5x^2 \text{ and } 5x^3 \quad 9ab \text{ and } 0.4a \quad \frac{8}{3}x^3y^2 \text{ and } \frac{4}{7}x^2y^3$$

To add like terms, we use the distributive property of multiplication over addition.

$$\begin{aligned} 9x + 2x &= (9 + 2)x = 11x \\ -16d + 3d &= (-16 + 3)d = -13d \end{aligned}$$

Note that in the above examples, when like terms are added:

1. The sum has the same variable factor as the original terms.
2. The numerical coefficient of the sum is the sum of the numerical coefficients of the terms that were added.

The sum of like terms can be expressed as a single term. The sum of unlike terms cannot be expressed as a single term. For example, the sum of $2x$ and 3 cannot be written as a single term but is written $2x + 3$.

EXAMPLE 1

Solve and check: $2x + 3x + 4 = -6$

Solution

How to Proceed

Check

- | | | |
|---|--------------------------------------|--|
| (1) Write the equation: | $2x + 3x + 4 = -6$ | $2x + 3x + 4 = -6$ |
| (2) Simplify the left side by combining like terms: | $5x + 4 = -6$ | $2(-2) + 3(-2) + 4 \stackrel{?}{=} -6$ |
| (3) Add -4 , the additive inverse of $+4$, to each side: | $\frac{-4}{5x} = \frac{-4}{-10}$ | $-4 - 6 + 4 \stackrel{?}{=} -6$
$-6 = -6$ ✓ |
| (4) Multiply by $\frac{1}{5}$, the multiplicative inverse of 5 : | $\frac{1}{5}(5x) = \frac{1}{5}(-10)$ | |
| (5) Simplify each side. | $x = -2$ | |

Answer -2



Note: When solving equations, remember to check the answer in the original equation and not in the simplified one.

The algebraic expression that is on one side of an equation may contain parentheses. Use the distributive property to remove the parentheses solving the equation. The following examples illustrate how the distributive and associative properties are used to do this.

EXAMPLE 2

Solve and check: $27x - 3(x - 6) = 6$

Solution Since $-3(x - 6)$ means that $(x - 6)$ is to be multiplied by -3 , we will use the distributive property to remove parentheses and then combine like terms. Note that for this solution, in the first three steps the left side is being simplified. These steps apply only to the left side and only change the form but not the numerical value. The next two steps undo the operations of addition and multiplication that make up the expression $24x + 18$. Since adding -18 and dividing by 24 will change the value of the left side, the right side must be changed in the same way to retain the equality.

How to Proceed

- | | |
|--|--|
| (1) Write the equation: | $27x - 3(x - 6) = 6$ |
| (2) Use the distributive property: | $27x - 3x + 18 = 6$ |
| (3) Combine like terms: | $24x + 18 = 6$ |
| (4) Use the addition property of equality. Add -18 , the additive inverse of $+18$, to each side: | $\begin{array}{r} 24x + 18 = 6 \\ -18 \quad -18 \\ \hline 24x = -12 \end{array}$ |
| (5) Use the division property of equality. Divide each side by 24 : | $\frac{24x}{24} = \frac{-12}{24}$ |
| (6) Simplify each side: | $x = -\frac{1}{2}$ |

Check

- | | |
|--|---|
| (1) Write the equation: | $27x - 3(x - 6) = 6$ |
| (2) Replace x by $-\frac{1}{2}$: | $27\left(-\frac{1}{2}\right) - 3\left(-\frac{1}{2} - 6\right) \stackrel{?}{=} 6$ |
| (3) Perform the indicated computation: | $\begin{aligned} 27\left(-\frac{1}{2}\right) - 3\left(-6\frac{1}{2}\right) &\stackrel{?}{=} 6 \\ -\frac{27}{2} + 18\frac{3}{2} &\stackrel{?}{=} 6 \\ -\frac{27}{2} + \frac{39}{2} &\stackrel{?}{=} 6 \\ \frac{12}{2} &\stackrel{?}{=} 6 \\ 6 &= 6 \quad \checkmark \end{aligned}$ |

Answer $x = -\frac{1}{2}$

Representing Two Numbers with the Same Variable

Problems often involve finding two or more different numbers. It is useful to express these numbers in terms of the same variable. For example, if you know the sum of two numbers, you can express the second in terms of the sum and the first number.

- If the sum of two numbers is 12 and one of the numbers is 5, then the other number is $12 - 5$ or 7.
- If the sum of two numbers is 12 and one of the numbers is 9, then the other number is $12 - 9$ or 3.
- If the sum of two numbers is 12 and one of the numbers is x , then the other number is $12 - x$.

A problem can often be solved algebraically in more than one way by writing and solving different equations, as shown in the example that follows. The methods used to obtain the solution are different, but both use the facts stated in the problem and arrive at the same solution.

EXAMPLE 3

The sum of two numbers is 43. The larger number minus the smaller number is 5. Find the numbers.

Solution This problem states two facts:

FACT 1 The sum of the numbers is 43.

FACT 2 The larger number minus the smaller number is 5. In other words, the larger number is 5 more than the smaller.

- (1) Represent each number in terms of the same variable using Fact 1:
the sum of the numbers is 43.

Let x = the larger number.

Then, $43 - x$ = the smaller number.

- (2) Write an equation using Fact 2:

$$\begin{array}{ccccccc} \text{The larger number} & \text{minus} & \text{the smaller number} & \text{is} & 5. \\ \hline x & - & (43 - x) & = & 5 \end{array}$$

(3) Solve the equation.

(a) Write the equation:

$$x - (43 - x) = 5$$

(b) To subtract $(43 - x)$, add its opposite:

$$x + (-43 + x) = 5$$

(c) Combine like terms:

$$2x - 43 = 5$$

(d) Add the opposite of -43 to each side:

$$\frac{2x}{2x} \begin{array}{r} +43 \\ +43 \end{array} = \frac{5}{48} \begin{array}{r} +43 \\ +43 \end{array}$$

(e) Divide each side by 2:

$$\frac{2x}{2} = \frac{48}{2}$$

$$x = 24$$

(4) Find the numbers.

The larger number = $x = 24$.

The smaller number = $43 - x = 43 - 24 = 19$.

Check A word problem is checked by comparing the proposed solution with the facts stated in the original wording of the problem. Substituting numbers in the equation is not sufficient since the equation formed may not be correct.

The sum of the numbers is 43: $24 + 19 = 43$.

The larger number minus the smaller number is 5: $24 - 19 = 5$.

Alternate Solution Reverse the way in which the facts are used.

(1) Represent each number in terms of the same variable using Fact 2:

the larger number is 5 more than the smaller.

Let $x =$ the smaller number.

Then, $x + 5 =$ the larger number.

(2) Write an equation using the first fact.

The sum of the numbers is 43.

$$\begin{array}{r} \underline{x + (x + 5)} \\ \downarrow \quad \downarrow \\ = 43 \end{array}$$

(3) Solve the equation.

(a) Write the equation:

$$x + (x + 5) = 43$$

(b) Combine like terms:

$$2x + 5 = 43$$

(c) Add the opposite of 5 to each side:

$$\frac{2x}{2x} \begin{array}{r} -5 \\ -5 \end{array} = \frac{38}{38}$$

(d) Divide each side by 2:

$$\frac{2x}{2} = \frac{38}{2}$$

$$x = 19$$

(4) Find the numbers.

The smaller number = $x = 19$.

The larger number = $x + 5 = 19 + 5 = 24$.

(5) **Check.** (See the first solution.)

Answer The numbers are 24 and 19. ■

EXERCISES**Writing About Mathematics**

- Two students are each solving a problem that states that the difference between two numbers is 12. Irene represents one number by x and the other number by $x + 12$. Henry represents one number by x and the other number by $x - 12$. Explain why both students are correct.
- A problem states that the sum of two numbers is 27. The numbers can be represented by x and $27 - x$. Is it possible to determine which is the larger number and which is the smaller number? Explain your answer.

Developing Skills

In 3–28, solve and check each equation.

- | | |
|--|---|
| 3. $x + (x - 6) = 20$ | 4. $x - (12 - x) = 38$ |
| 5. $(15x + 7) - 12 = 4$ | 6. $(14 - 3c) + 7c = 94$ |
| 7. $x + (4x + 32) = 12$ | 8. $7x - (4x - 39) = 0$ |
| 9. $5(x + 2) = 20$ | 10. $3(y - 9) = 30$ |
| 11. $8(2c - 1) = 56$ | 12. $6(3c - 1) = -42$ |
| 13. $30 = 2(10 - y)$ | 14. $4(c + 1) = 32$ |
| 15. $25 - 2(t - 5) = 19$ | 16. $18 = -6x + 4(2x + 3)$ |
| 17. $55 = 4 + 3(m + 2)$ | 18. $5(x - 3) - 30 = 10$ |
| 19. $3(2b + 1) - 7 = 50$ | 20. $5(3c - 2) + 8 = 43$ |
| 21. $7r - (6r - 5) = 7$ | 22. $8b - 4(b - 2) = 24$ |
| 23. $5m - 2(m - 5) = 17$ | 24. $28y - 6(3y - 5) = 40$ |
| 25. $3(a - 5) - 2(2a + 1) = 0$ | 26. $0.04(2r + 1) - 0.03(2r - 5) = 0.29$ |
| 27. $0.3a + (0.2a - 0.5) + 0.2(a + 2) = 1.3$ | 28. $\frac{3}{4}(8 + 4x) - \frac{1}{3}(6x + 3) = 9$ |

Applying Skills

In 29–33, write and solve an equation for each problem. Follow these steps:

- List two facts in the problem.
- Choose a variable to represent one of the numbers to be determined.
- Use one of the facts to write any other unknown numbers in terms of the chosen variable.
- Use the second fact to write an equation.
- Solve the equation.

- f. Answer the question.
- g. Check your answer using the words of the problem.
29. Sandi bought 6 yards of material. She wants to cut it into two pieces so that the difference between the lengths of the two pieces will be 1.5 yards. What should be the length of each piece?
30. The Tigers won eight games more than they lost, and there were no ties. If the Tigers played 78 games, how many games did they lose?
31. This month Erica saved \$20 more than last month. For the two months, she saved a total of \$70. How much did she save each month?
32. On a bus tour, there are 100 passengers on three buses. Two of the buses each carry four fewer passengers than the third bus. How many passengers are on each bus?
33. For a football game, $\frac{4}{5}$ of the seats in the stadium were filled. There were 31,000 empty seats at the game. What is the stadium's seating capacity?

4-3 SOLVING EQUATIONS THAT HAVE THE VARIABLE IN BOTH SIDES

A variable represents a number. As you know, any number may be added to both sides of an equation without changing the solution set. Therefore, the same variable (or the same multiple of the same variable) may be added to or subtracted from both sides of an equation without changing the solution set.

For instance, to solve $8x = 30 + 5x$, write an equivalent equation that has only a constant in the right side. To do this, eliminate $5x$ from the right side by adding its opposite, $-5x$, to each side of the equation.

METHOD 1	METHOD 2	<i>Check</i>
$8x = 30 + 5x$	$8x = 30 + 5x$	$8x = 30 + 5x$
$\frac{-5x}{3x} = \frac{\quad -5x}{30}$	$8x + (-5x) = 30 + 5x + (-5x)$	$8(10) \stackrel{?}{=} 30 + 5(10)$
$3x = 30$	$3x = 30$	$80 \stackrel{?}{=} 30 + 50$
$x = 10$	$x = 10$	$80 = 80 \checkmark$

Answer: $x = 10$

To solve an equation that has the variable in both sides, transform it into an equivalent equation in which the variable appears in only one side. Then, solve the equation.

EXAMPLE 1Solve and check: $7x = 63 - 2x$ **Solution***How to Proceed*

- | | |
|--|--|
| (1) Write the equation: | $7x = 63 - 2x$ |
| (2) Add $2x$ to each side of the equation: | $\begin{array}{r} 7x = 63 - 2x \\ +2x \quad \quad +2x \\ \hline 9x = 63 \end{array}$ |
| (3) Divide each side of the equation by 9: | $\frac{9x}{9} = \frac{63}{9}$ |
| (4) Simplify each side: | $x = 7$ |

Check

$$\begin{aligned} 7x &= 63 - 2x \\ 7(7) &\stackrel{?}{=} 63 - 2(7) \\ 49 &\stackrel{?}{=} 63 - 14 \\ 49 &= 49 \quad \checkmark \end{aligned}$$

Answer $x = 7$ ■

To solve an equation that has both a variable and a constant in both sides, first write an equivalent equation with only a variable term on one side. Then solve the simplified equation. The following example shows how this can be done.

EXAMPLE 2Solve and check: $3y + 7 = 5y - 3$ **Solution****METHOD 1**

$$\begin{aligned} 3y + 7 &= 5y - 3 \\ \frac{-5y}{-2y + 7} &= \frac{-5y}{-3} \\ \frac{-7}{-2y} &= \frac{-7}{-10} \\ \frac{-2y}{-2} &= \frac{-10}{-2} \\ y &= 5 \end{aligned}$$

METHOD 2

$$\begin{aligned} 3y + 7 &= 5y - 3 \\ \frac{-3y}{7} &= \frac{-3y}{2y - 3} \\ \frac{+3}{10} &= \frac{+3}{2y} \\ \frac{10}{2} &= \frac{2y}{2} \\ y &= 5 \end{aligned}$$

Check

$$\begin{aligned} 3y + 7 &= 5y - 3 \\ 3(5) + 7 &\stackrel{?}{=} 5(5) - 3 \\ 15 + 7 &\stackrel{?}{=} 25 - 3 \\ 22 &= 22 \quad \checkmark \end{aligned}$$

Answer $y = 5$ ■

A graphing calculator can be used to check an equation. The calculator can determine whether a given statement of equality or inequality is true or false. If the statement is true, the calculator will display 1; if the statement is false, the calculator will display 0. The symbols for equality and inequality are found in the **TEST** menu.

To check that $y = 5$ is the solution to the equation $3y + 7 = 5y - 3$, first store 5 as the value of y , then enter the equation to be checked.

ENTER: 5 **STO** **ALPHA** **Y** **ENTER**

3 **ALPHA** **Y** **+** 7 **2nd** **TEST** **ENTER** 5 **ALPHA** **Y** **-** 3 **ENTER**

DISPLAY:

$5 \rightarrow Y$ $3Y + 7 = 5Y - 3$	5 1
--	----------------

The calculator displays 1 which indicates that the statement of equality is true for the value that has been stored for y .

EXAMPLE 3

The larger of two numbers is 4 times the smaller. If the larger number exceeds the smaller number by 15, find the numbers.

Note: When s represents the smaller number and $4s$ represents the larger number, “the larger number *exceeds* the smaller by 15” has the following meanings. Use any one of them.

1. The larger equals 15 more than the smaller, written as $4s = 15 + s$.
2. The larger decreased by 15 equals the smaller, written as $4s - 15 = s$.
3. The larger decreased by the smaller is 15, written as $4s - s = 15$.

Solution Let s = the smaller number.

Then $4s$ = the larger number.

The larger is 15 more than the smaller.

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 4s & = & 15 & + & s & & \end{array}$$

$$4s = 15 + s$$

$$4s = 15 + s$$

$$\begin{array}{r} -s \quad \quad -s \\ \hline 3s = 15 \end{array}$$

$$s = 5$$

$$4s = 4(5) = 20$$

Check The larger number, 20, is 4 times the smaller number, 5. The larger number, 20, exceeds the smaller number, 5, by 15.

Answer The larger number is 20; the smaller number is 5. ■

EXAMPLE 4

In his will, Uncle Clarence left \$5,000 to his two nieces. Emma's share is to be \$500 more than Clara's. How much should each niece receive?

- Solution** (1) Use the fact that the sum of the two shares is \$5,000 to express each share in terms of a variable.
Let x = Clara's share.
Then $5,000 - x$ = Emma's share.
- (2) Use the fact that Emma's share is \$500 more than Clara's share to write an equation.

$$\begin{array}{ccccccc} \text{Emma's share is } \$500 \text{ more than } & & & & \text{Clara's share.} & & \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 5,000 - x & = & 500 & + & & & x \end{array}$$

- (3) Solve the equation to find Clara's share.

$$\begin{array}{r} 5,000 - x = 500 + x \\ \quad \quad + x \quad \quad \quad + x \\ \hline 5,000 \quad \quad = \quad 500 + 2x \\ -500 \quad \quad \quad -500 \\ \hline 4,500 \quad \quad = \quad \quad \quad 2x \end{array}$$

$$2,250 = x$$

Clara's share is $x = \$2,250$.

- (4) Find Emma's share: $5,000 - x = 5,000 - 2,250 = \$2,750$.

- Alternate Solution** (1) Use the fact that Emma's share is \$500 more than Clara's share to express each share in terms of a variable.
Let x = Clara's share.
Then $x + 500$ = Emma's share.

- (2) Use the fact that the sum of the two shares is \$5,000 to write an equation.

$$\begin{array}{ccccccc} \text{Clara's share plus Emma's share is } \$5,000. & & & & & & \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ x & + & (x + 500) & = & 5,000 \end{array}$$

- (3) Solve the equation to find Clara's share

$$\begin{array}{r} x + (x + 500) = 5,000 \\ 2x + 500 = 5,000 \\ \quad \quad -500 \quad \quad -500 \\ \hline 2x \quad \quad = \quad 4,500 \\ \quad \quad \quad x \quad \quad = \quad 2,250 \end{array}$$

Clara's share is $x = \$2,250$.

(4) Find Emma's share: $x + 500 = 2250 + 500 = \$2,750$.

Check \$2,750 is \$500 more than \$2,250, and $\$2,750 + \$2,250 = \$5,000$.

Answer Clara's share is \$2,250, and Emma's share is \$2,750. ■

EXERCISES

Writing About Mathematics

1. Milus said that he finds it easier to work with integers than with fractions. Therefore, in order to solve the equation $\frac{3}{4}a - 7 = \frac{1}{2}a + 3$, he began by multiplying both sides of the equation by 4.

$$\begin{aligned} 4\left(\frac{3}{4}a - 7\right) &= 4\left(\frac{1}{2}a + 3\right) \\ 3a - 28 &= 2a + 12 \end{aligned}$$

Do you agree with Milus that this is a correct way of obtaining the solution? If so, what mathematical principle is Milus using?

2. Katie said that Example 3 could be solved by letting $\frac{x}{4}$ equal the smaller number and x equal the larger number. Is Katie correct? If so, what equation would she write to solve the problem?

Developing Skills

In 3–36, solve and check each equation.

- | | | |
|--|----------------------------------|--|
| 3. $7x = 10 + 2x$ | 4. $9x = 44 - 2x$ | 5. $5c = 28 + c$ |
| 6. $y = 4y + 30$ | 7. $2d = 36 + 5d$ | 8. $2\frac{1}{4}y = 1\frac{1}{4}y - 8$ |
| 9. $0.8m = 0.2m + 24$ | 10. $8y = 90 - 2y$ | 11. $2.3x + 36 = 0.3x$ |
| 12. $2\frac{3}{4}x + 24 = 3x$ | 13. $5a - 40 = 3a$ | 14. $5c = 2c - 81$ |
| 15. $x = 9x - 72$ | 16. $0.5m - 30 = 1.1m$ | 17. $4\frac{1}{4}c = 9\frac{3}{4}c + 44$ |
| 18. $7r + 10 = 3r + 50$ | 19. $4y + 20 = 5y + 9$ | 20. $7x + 8 = 6x + 1$ |
| 21. $x + 4 = 9x + 4$ | 22. $9x - 3 = 2x + 46$ | 23. $y + 30 = 12y - 14$ |
| 24. $c + 20 = 55 - 4c$ | 25. $2d + 36 = -3d - 54$ | 26. $7y - 5 = 9y + 29$ |
| 27. $3m - (m + 1) = 6m + 1$ | 28. $x - 3(1 - x) = 47 - x$ | 29. $3b - 8 = 10 + (4 - 8b)$ |
| 30. $\frac{2}{3}t - 11 = 4(16 - t) - \frac{1}{3}t$ | 31. $18 - 4n = 8 - 2(1 + 8n)$ | |
| 32. $8c + 1 = 7c - 2(7 + c)$ | 33. $8a - 3(5 + 2a) = 85 - 3a$ | |
| 34. $4(3x - 5) = 5x + 2(x + 15)$ | 35. $3m - 5m - 12 = 7m - 88 - 5$ | |
| 36. $5 - 3(a + 6) = a - 1 + 8a$ | | |

In 37–42, **a.** write an equation to represent each problem, and **b.** solve the equation to find each number.

37. Eight times a number equals 35 more than the number. Find the number.
38. Six times a number equals 3 times the number, increased by 24. Find the number.
39. If 3 times a number is increased by 22, the result is 14 less than 7 times the number. Find the number.
40. The greater of two numbers is 1 more than twice the smaller. Three times the greater exceeds 5 times the smaller by 10. Find the numbers.
41. The second of three numbers is 6 more than the first. The third number is twice the first. The sum of the three numbers is 26. Find the three numbers.
42. The second of three numbers is 1 less than the first. The third number is 5 less than the second. If the first number is twice as large as the third, find the three numbers.

Applying Skills

In 43–50, use an algebraic solution to solve each problem.

43. It took the Gibbons family 2 days to travel 925 miles to their vacation home. They traveled 75 miles more on the first day than on the second. How many miles did they travel each day?
44. During the first 6 months of last year, the interest on an investment was \$130 less than during the second 6 months. The total interest for the year was \$1,450. What was the interest for each 6-month period?
45. Gemma has 7 more five-dollar bills than ten-dollar bills. The value of the five-dollar bills equals the value of the ten-dollar bills. How many five-dollar bills and ten-dollar bills does she have?
46. Leonard wants to save \$100 in the next 2 months. He knows that in the second month he will be able to save \$20 more than during the first month. How much should he save each month?
47. The ABC Company charges \$75 a day plus \$0.05 a mile to rent a car. How many miles did Mrs. Kiley drive if she paid \$92.40 to rent a car for one day?
48. Kesha drove from Buffalo to Syracuse at an average rate of 48 miles per hour. On the return trip along the same road she was able to travel at an average rate of 60 miles per hour. The trip from Buffalo to Syracuse took one-half hour longer than the return trip. How long did the return trip take?
49. Carrie and Crystal live at equal distances from school. Carrie walks to school at an average rate of 3 miles per hour and Crystal rides her bicycle at an average rate of 9 miles per hour. It takes Carrie 20 minutes longer than Crystal to get to school. How far from school do Crystal and Carrie live?
50. Emmanuel and Anthony contributed equal amounts to the purchase of a gift for a friend. Emmanuel contributed his share in five-dollar bills and Anthony gave his share in one-dollar bills. Anthony needed 12 more bills than Emmanuel. How much did each contribute toward the gift?

4-4 USING FORMULAS TO SOLVE PROBLEMS

To solve for the subject of a formula, substitute the known values in the formula and perform the required computation. For example, to find the area of a triangle when $b = 4.70$ centimeters and $h = 3.20$ centimeters, substitute the given values in the formula for the area of a triangle:

$$\begin{aligned} A &= \frac{1}{2}bh && \text{A is the subject of the formula.} \\ &= \frac{1}{2}(4.70 \text{ cm})(3.20 \text{ cm}) \\ &= 7.52 \text{ cm}^2 \end{aligned}$$

Now that you can solve equations, you will be able to find the value of any variable in a formula when the values of the other variables are known. To do this:

1. Write the formula.
2. Substitute the given values in the formula.
3. Solve the resulting equation.

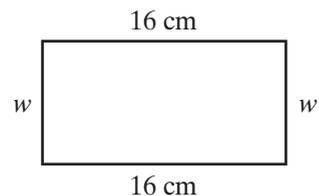
The values assigned to the variables in a formula often have a unit of measure. It is convenient to solve the equation without writing the unit of measure, but the answer should always be given in terms of the correct unit of measure.

EXAMPLE 1

The perimeter of a rectangle is 48 centimeters. If the length of the rectangle is 16 centimeters, find the width to the nearest centimeter.

Solution You know that the **perimeter** of a geometric figure is the sum of the lengths of all of its sides. When solving a perimeter problem, it is helpful to draw and label a figure to model the region. Use the formula $P = 2l + 2w$.

$$\begin{aligned} P &= 2l + 2w && \text{Check} \\ 48 &= 2(16) + 2w && P = 2l + 2w \\ 48 &= 32 + 2w && 48 \stackrel{?}{=} 2(16) + 2(8) \\ \frac{-32}{16} &= \frac{-32}{2w} && 48 \stackrel{?}{=} 32 + 16 \\ 16 &= 2w && 48 = 48 \checkmark \\ 8 &= w \end{aligned}$$



Answer 8 centimeters

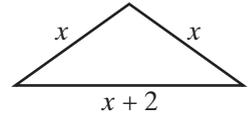


EXAMPLE 2

A garden is in the shape of an isosceles triangle, a triangle that has two sides of equal measure. The length of the third side of the triangle is 2 feet greater than the length of each of the equal sides. If the perimeter of the garden is 86 feet, find the length of each side of the garden.

Solution Let x = the length of each of the two equal sides.

Then, $x + 2$ = the length of the third side.



The perimeter is the sum of the lengths of the sides.

$$\begin{array}{ccccccc} \downarrow & & \downarrow & & \downarrow & & \\ 86 & = & x & + & x & + & (x + 2) \end{array}$$

$$86 = 3x + 2$$

$$84 = 3x$$

$$28 = x$$

The length of each of the equal sides = $x = 28$.

The length of the third side = $x + 2 = 28 + 2 = 30$.

Check Perimeter = $28 + 28 + 30 = 86$ ✓

Answer The length of each of the equal sides is 28 feet. The length of the third side (the base) is 30 feet. ■

EXAMPLE 3

The perimeter of a rectangle is 52 feet. The length is 2 feet more than 5 times the width. Find the dimensions of the rectangle.

Solution Use the formula for the perimeter of a rectangle, $P = 2l + 2w$, to solve this problem.

Let w = the width, in feet, of the rectangle.

Then $5w + 2$ = the length, in feet, of the rectangle.

$$P = 2l + 2w$$

Check

$$52 = 2(5w + 2) + 2w$$

The length, 22, is 2 more than 5 times the width, 4. ✓

$$52 = 10w + 4 + 2w$$

$$P = 2l + 2w$$

$$52 = 12w + 4$$

$$52 \stackrel{?}{=} 2(22) + 2(4)$$

$$48 = 12w$$

$$52 \stackrel{?}{=} 44 + 8$$

$$4 = w$$

$$52 = 52 \quad \checkmark$$

Answer The width is 4 feet; the length is $5(w) + 2 = 5(4) + 2 = 22$ feet. ■

EXAMPLE 4

Sabrina drove from her home to her mother's home which is 150 miles away. For the first half hour, she drove on local roads. For the next two hours she drove on an interstate highway and increased her average speed by 15 miles per hour. Find Sabrina's average speed on the local roads and on the interstate highway.

Solution List the facts stated by this problem:

FACT 1 Sabrina drove on local roads for $\frac{1}{2}$ hour or 0.5 hour.

FACT 2 Sabrina drove on the interstate highway for 2 hours.

FACT 3 Sabrina's rate or speed on the interstate highway was 15 mph more than her rate on local roads.

This problem involves rate, time, and distance. Use the **distance formula**, $d = rt$, where r is the rate, or speed, in miles per hour, t is time in hours, and d is distance in miles.

(1) Represent Sabrina's speed for each part of the trip in terms of r .

Let r = Sabrina's speed on the local roads.

Then $r + 15$ = Sabrina's speed on the interstate highway.

(2) Organize the facts in a table, using the distance formula.

	Rate \times Time = Distance		
Local Roads	r	0.5	$0.5r$
Interstate highway	$r + 15$	2	$2(r + 15)$

(3) Write an equation.

The distance on the local roads plus the distance on the highway is 150 miles.

$$\begin{array}{ccccccc} \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 0.5r & + & 2(r + 15) & = & 150 \end{array}$$

(4) Solve the equation.

(a) Write the equation:

$$0.5r + 2(r + 15) = 150$$

(b) Use the distributive property:

$$0.5r + 2r + 30 = 150$$

(c) Combine like terms:	$2.5r + 30 = 150$
(d) Add -30 , the opposite of $+30$ to each side of the equation:	$\begin{array}{r} 2.5r + 30 = 150 \\ -30 \quad -30 \\ \hline 2.5r = 120 \end{array}$
(e) Divide each side by 2.5:	$\begin{array}{r} 2.5r = 120 \\ \hline r = 48 \end{array}$

(5) Find the average speed for each part of the trip.

Sabrina's speed on local roads = $r = 48$ mph.

Sabrina's speed on the highway = $r + 15 = 48 + 15 = 63$ mph.

Check On local roads: $0.5(48) = 24$ miles
 On the interstate highway: $2(63) = \underline{126}$ miles
 The total distance traveled: 150 miles ✓

Answer Sabrina traveled at an average speed of 48 miles per hour on local roads and 63 miles per hour on the interstate highway. ▣

EXERCISES

Writing About Mathematics

- In Example 4, step 4 uses the equivalent equation $2.5r + 30 = 150$. Explain what each term in the left side of this equation represents in the problem.
- Antonio solved the problem in Example 4 by letting r represent Sabrina's rate of speed on the interstate highway. To complete the problem correctly, how should Antonio represent her rate of speed on the local roads? Explain your answer.

Developing Skills

In 3–19, state the meaning of each formula, tell what each variable represents, and find the required value. In 3–12, express each answer to the correct number of significant digits.

- If $P = a + b + c$, find c when $P = 85$ in., $a = 25$ in., and $b = 12$ in.
- If $P = 4s$, find s when $P = 32$ m.
- If $P = 4s$, find s when $P = 6.8$ ft.
- If $P = 2l + 2w$, find w when $P = 26$ yd and $l = 8$ yd.
- If $P = 2a + b$, find b when $P = 80$ cm and $a = 30$ cm.
- If $P = 2a + b$, find a when $P = 18.6$ m and $b = 5.8$ m.
- If $A = bh$, find b when $A = 240$ cm² and $h = 15$ cm.

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10. If $A = bh$, find h when $A = 3.6 \text{ m}^2$ and $b = 0.90 \text{ m}$.
11. If $A = \frac{1}{2}bh$, find h when $A = 24 \text{ sq ft}$ and $b = 8.0 \text{ ft}$.
12. If $V = lwh$, find w when $V = 72 \text{ yd}^3$, $l = 0.75 \text{ yd}$, and $h = 12 \text{ yd}$.
13. If $d = rt$, find r when $d = 120 \text{ mi}$ and $t = 3 \text{ hr}$.
14. If $I = prt$, find the principal, p , when the interest, I , is \$135, the yearly rate of interest, r , is 2.5%, and the time, t , is 3 years.
15. If $I = prt$, find the rate of interest, r , when $I = \$225$, $p = \$2,500$, and $t = 2$ years.
16. If $T = nc$, find the number of items purchased, n , if the total cost, T , is \$19.80 and the cost of one item, c , is \$4.95.
17. If $T = nc$, find the cost of one item purchased, c , if $T = \$5.88$ and $n = 12$.
18. If $S = nw$, find the hourly wage, w , if the salary earned, S , is \$243.20 and the number of hours worked, n , is 38.
19. If $S = nw$, find the number of hours worked, n , if $S = \$315.00$ and $w = \$8.40$.

In 20–32, **a.** write a formula that can be used to solve each problem, **b.** use the formula to solve each problem and check the solution. All numbers may be considered to be exact values.

20. Find the length of a rectangle whose perimeter is 34.6 centimeters and whose width is 5.7 centimeters.
21. The length of the second side of a triangle is 2 inches less than the length of the first side. The length of the third side is 12 inches more than the length of the first side. The perimeter of the triangle is 73 inches. Find the length of each side of the triangle.
22. Two sides of a triangle are equal in length. The length of the third side exceeds the length of one of the other sides by 3 centimeters. The perimeter of the triangle is 93 centimeters. Find the length of each side of the triangle.
23. The length of a rectangle is 5 meters more than its width. The perimeter is 66 meters. Find the dimensions of the rectangle.
24. The width of a rectangle is 3 yards less than its length. The perimeter is 130 yards. Find the length and the width of the rectangle.
25. The length of each side of an equilateral triangle is 5 centimeters more than the length of each side of a square. The perimeters of the two figures are equal. Find the lengths of the sides of the square and of the triangle.
26. The length of each side of a square is 1 centimeter more than the width of a rectangle. The length of the rectangle is 1 centimeter less than twice its width. The perimeters of the two figures are equal. Find the dimensions of the rectangle.
27. The area of a triangle is 36 square centimeters. Find the measure of the altitude drawn to the base when the base is 8 centimeters.

28. The altitude of a triangle is 4.8 meters. Find the length of the base of the triangle if the area is 8.4 square meters.
29. The length of a rectangle is twice the width. If the length is increased by 4 inches and the width is decreased by 1 inch, a new rectangle is formed whose perimeter is 198 inches. Find the dimensions of the original rectangle.
30. The length of a rectangle exceeds its width by 4 feet. If the width is doubled and the length is decreased by 2 feet, a new rectangle is formed whose perimeter is 8 feet more than the perimeter of the original rectangle. Find the dimensions of the original rectangle.
31. A side of a square is 10 meters longer than the side of an equilateral triangle. The perimeter of the square is 3 times the perimeter of the triangle. Find the length of each side of the triangle.
32. The length of each side of a hexagon is 4 inches less than the length of a side of a square. The perimeter of the hexagon is equal to the perimeter of the square. Find the length of a side of the hexagon and the length of a side of the square.

Applying Skills

33. The perimeter of a rectangular parking lot is 146 meters. Find the dimensions of the lot, using the correct number of significant digits, if the length is 7.0 meters less than 4 times the width.
34. The perimeter of a rectangular tennis court is 228 feet. If the length of the court exceeds twice its width by 6.0 feet, find the dimensions of the court using the correct number of significant digits.

In 35–48, make a table to organize the information according to the formula to be used. All numbers may be considered to be exact values.

35. Rahul has 25 coins, all quarters and dimes. Copy the table given below and organize the facts in the table using the answers to **a** through **c**.

	$\left(\begin{array}{c} \text{Number of coins} \\ \text{in one denomination} \end{array} \right) \times \left(\begin{array}{c} \text{Value of} \\ \text{one coin} \end{array} \right) = \begin{array}{c} \text{Total value of the coins} \\ \text{of that denomination} \end{array}$		
	Number of Coins	Value of One Coin	Total Value
Dimes			
Quarters			

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- a. If x is the number of dimes Rahul has, express, in terms of x , the number of quarters he has.
- b. Express the value of the dimes in terms of x .
- c. Express the value of the quarters in terms of x .
- d. If the total value of the dimes and quarters is \$4.90, write and solve an equation to find how many dimes and how many quarters Rahul has.
- e. Check your answer in the words of the problem.
36. If the problem had said that the total value of Rahul's 25 dimes and quarters was \$5.00, what conclusion could you draw?
37. When Ruth emptied her bank, she found that she had 84 coins, all nickels and dimes. The value of the coins was \$7.15. How many dimes did she have? (Make a table similar to that given in exercise 35.)
38. Adele went to the post office to buy stamps and postcards. She bought a total of 25 stamps, some 39-cent stamps and the rest 23-cent postcards. If she paid \$8.47 altogether, how many 39-cent stamps did she buy?
39. Carlos works Monday through Friday and sometimes on Saturday. Last week Carlos worked 38 hours. Copy the table given below and organize the facts in the table using the answers to **a** through **c**.

	Hours Worked	Wage Per Hour	Earnings
Monday–Friday			
Saturday			

- a. If x is the total number of hours Carlos worked Monday through Friday, express, in terms of x , the number of hours he worked on Saturday.
- b. Carlos earns \$8.50 an hour when he works Monday through Friday. Express, in terms of x , his earnings Monday through Friday.
- c. Carlos earns \$12.75 an hour when he works on Saturday. Express, in terms of x , his earnings on Saturday.
- d. Last week Carlos earned \$340. How many hours did he work on Saturday?
40. Janice earns \$6.00 an hour when she works Monday through Friday and \$9.00 an hour when she works on Saturday. Last week, her salary was \$273 for 42 hours of work. How many hours did she work on Saturday? (Make a table similar to that given in exercise 39.)
41. Candice earns \$8.25 an hour and is paid every two weeks. Last week she worked 4 hours longer than the week before. Her pay for these two weeks, before deductions, was \$594. How many hours did she work each week?

42. Akram drove from Rochester to Albany, a distance of 219 miles. After the first 1.5 hours of travel, it began to snow and he reduced his speed by 26 miles per hour. It took him another 3 hours to complete the trip. Copy the table given below and fill in the entries using the answers to **a** through **c**.

	Rate	Time	Distance
First part of the trip			
Last part of the trip			

- If r is the average speed at which Akram traveled for the first part of the trip, express, in terms of r , his average speed for the second part of the trip.
 - Express, in terms of r , the distance that Akram traveled in the first part of the trip.
 - Express, in terms of r , the distance that Akram traveled in the second part of the trip.
 - Find the speed at which Akram traveled during each part of the trip.
43. Vera walked from her home to a friend's home at a rate of 3 miles per hour. She rode to work with her friend at an average rate of 30 miles per hour. It took Vera a total of 50 minutes ($\frac{5}{6}$ of an hour) to walk to her friend's home and to get to work, traveling a total distance of 16 miles. How long did she walk and how long did she ride with her friend to get to work? (Make a table similar to that given in exercise 42.)
44. Peter drove a distance of 189 miles. Part of the time he averaged 65 miles per hour and for the remaining time, 55 miles per hour. The entire trip took 3 hours. How long did he travel at each rate?
45. Shelly and Jack left from the same place at the same time and drove in opposite directions along a straight road. Jack traveled 15 miles per hour faster than Shelly. After 3 hours, they were 315 miles apart. Find the rate at which each traveled.
46. Carla and Candice left from the same place at the same time and rode their bicycles in the same direction along a straight road. Candice bicycled at an average speed that was three-quarters of Carla's average speed. After 2 hours they were 28 miles apart. What was the average speed of Carla and Candice?
47. Nolan walked to the store from his home at the rate of 5 miles per hour. After spending one-half hour in the store, his friend gave him a ride home at the rate of 30 miles per hour. He arrived home 1 hour and 5 minutes ($1\frac{1}{12}$ hours) after he left. How far is the store from Nolan's home?
48. Mrs. Dang drove her daughter to school at an average rate of 45 miles per hour. She returned home by the same route at an average rate of 30 miles per hour. If the trip took one-half hour, How long did it take to get to school? How far is the school from their home?

4-5 SOLVING FOR A VARIABLE IN TERMS OF ANOTHER VARIABLE

An equation may contain more than one variable. For example, the equation $ax + b = 3b$ contains the variables a , b , and x . To solve this equation for x means to express x in terms of the other variables.

To plan the steps in the solution, it is helpful to use the strategy of using a simpler related problem, that is, to compare the solution of this equation with the solution of a simpler equation that has only one variable. In Example 1, the solution of $ax + b = 3b$ is compared with the solution of $2x + 5 = 15$. The same operations are used in the solution of both equations.

EXAMPLE 1

Solve for x in $ax + b = 3b$.

Solution Compare with $2x + 5 = 15$.

$$\begin{array}{r} 2x + 5 = 15 \\ -5 \quad -5 \\ \hline 2x = 10 \\ \frac{2x}{2} = \frac{10}{2} \\ x = 5 \end{array}$$

$$\begin{array}{r} ax + b = 3b \\ -b \quad -b \\ \hline ax = 2b \\ \frac{ax}{a} = \frac{2b}{a} \\ x = \frac{2b}{a} \end{array}$$

Check

$$\begin{array}{l} ax + b = 3b \\ a\left(\frac{2b}{a}\right) + b \stackrel{?}{=} 3b \\ 2b + b \stackrel{?}{=} 3b \\ 3b = 3b \checkmark \end{array}$$

Answer $x = \frac{2b}{a}$

EXAMPLE 2

Solve for x in $x - a = b$.

Solution Compare with $x - 5 = 9$.

$$\begin{array}{r} x - 5 = 9 \\ +5 \quad +5 \\ \hline x = 14 \end{array}$$

$$\begin{array}{r} x - a = b \\ +a \quad +a \\ \hline x = b + a \end{array}$$

Check

$$\begin{array}{l} x - a = b \\ b + a - a \stackrel{?}{=} b \\ b = b \checkmark \end{array}$$

Answer $x = b + a$

EXAMPLE 3

Solve for x in $2ax = 10a^2 - 3ax$ ($a \neq 0$).

Solution Compare with $2x = 10 - 3x$.

$$\begin{array}{l|l} \frac{2x}{+3x} = \frac{10-3x}{+3x} & \frac{2ax}{+3ax} = \frac{10a^2-3ax}{+3ax} \\ \frac{5x}{5} = \frac{10}{5} & \frac{5ax}{5a} = \frac{10a^2}{5a} \\ x = 2 & x = 2a \end{array}$$

Answer $x = 2a$

Check

$$\begin{aligned} 2ax &= 10a^2 - 3ax \\ 2a(2a) &\stackrel{?}{=} 10a^2 - 3a(2a) \\ 4a^2 &\stackrel{?}{=} 10a^2 - 6a^2 \\ 4a^2 &= 4a^2 \checkmark \end{aligned}$$

EXERCISES

Writing About Mathematics

- Write a simpler related equation in one variable that can be used to suggest the steps needed to solve the equation $a(x + b) = 4ab$ for x .
- Write a simpler related equation in one variable that can be used to suggest the steps needed to solve the equation $5cy = d + 2cy$ for y .

Developing Skills

In 3–24, solve each equation for x or y and check.

- | | | | |
|---------------------------|-----------------------------|--------------------------------|--------------------|
| 3. $5x = b$ | 4. $sx = 8$ | 5. $ry = s$ | 6. $hy = m$ |
| 7. $x + 5r = 7r$ | 8. $x + a = 4a$ | 9. $y + c = 9c$ | 10. $4 + x = k$ |
| 11. $d + y = 9$ | 12. $3x - q = 5q$ | 13. $3x - 8r = r$ | 14. $cy - d = 4d$ |
| 15. $ax + b = 3b$ | 16. $dx - 5c = 3c$ | 17. $r + sy = t$ | 18. $m = 2(x + n)$ |
| 19. $bx = 9b^2$ | 20. $cx + c^2 = 5c^2 - 7cx$ | 21. $rsx - rs^2 = 0$ | |
| 22. $m^2x - 3m^2 = 12m^2$ | 23. $9x - 24a = 6a + 4x$ | 24. $8ax - 7a^2 = 19a^2 - 5ax$ | |

4-6 TRANSFORMING FORMULAS

A formula is an equation that contains more than one variable. Sometimes you want to solve for a variable in the formula that is different from the subject of the formula. For example, the formula for distance, d , in terms of rate, r , and time, t , is $d = rt$. Distance is the subject of the formula, but you might want to rewrite the formula so that it expresses time in terms of distance and rate. You do this by solving the equation $d = rt$ for t in terms of d and r .

EXAMPLE 1

- a. Solve the formula $d = rt$ for t .
 b. Use the answer obtained in part a to find the value of t when $d = 200$ miles and $r = 40$ miles per hour.

Solution

$d = rt$ $\frac{d}{r} = \frac{rt}{r}$ $\frac{d}{r} = t$	$b. t = \frac{d}{r}$ $= \frac{200}{40}$ $= 5$
---	---

Answers a. $t = \frac{d}{r}$ b. $t = 5$ hours ■

Note that the rate is 40 miles per hour, that is, $\frac{40 \text{ miles}}{1 \text{ hour}}$. Therefore,

$$200 \text{ miles} \div \frac{40 \text{ miles}}{1 \text{ hour}} = 200 \text{ miles} \times \frac{1 \text{ hour}}{40 \text{ miles}} = 5 \text{ hours}$$

We can think of canceling miles in the numerator and the denominator of the fractions being multiplied.

EXAMPLE 2

- a. The formula for the volume of a cone is $V = \frac{1}{3}Bh$. Solve this formula for h .
 b. Find the height of a cone that has a volume of 92.0 cubic centimeters and a circular base with a radius of 2.80 centimeters. Express the answer using the correct number of significant digits.

Solution

$a.$	$V = \frac{1}{3}Bh$ $3V = 3\left(\frac{1}{3}Bh\right)$ $3V = Bh$ $\frac{3V}{B} = \frac{Bh}{B}$ $\frac{3V}{B} = h$
------	---

- b. Find B , the area of the base of the cone. Since the base is a circle, its area is π times the square of the radius, r .

$$B = \pi r^2$$

$$= \pi(2.80)^2$$

ENTER: 2nd π × 2.80 x^2 ENTER

DISPLAY: $\pi \cdot 2.80^2$
24.6300864

Now use the answer to part **a** to find h : $h = \frac{3V}{B} \approx \frac{3(92.0)}{24.63}$

ENTER: 3 \times 92.0 \div 24.63 **ENTER**

DISPLAY: $3 \cdot 92.0 / 24.63$
11.20584653

Since each measure is given to three significant digits, round the answer to three significant digits.

Answers **a.** $h = \frac{3V}{B}$ **b.** The height of the cone is 11.2 centimeters. ■

EXERCISES

Developing Skills

In 1–14, transform each given formula by solving for the indicated variable.

- | | | |
|-------------------------------------|--------------------------------|--------------------------------|
| 1. $P = 4s$ for s | 2. $A = bh$ for h | 3. $d = rt$ for r |
| 4. $V = lwh$ for l | 5. $P = br$ for r | 6. $I = prt$ for t |
| 7. $A = \frac{1}{2}bh$ for h | 8. $V = \frac{1}{3}Bh$ for B | 9. $s = \frac{1}{2}gt$ for g |
| 10. $P = 2a + b$ for b | 11. $P = 2a + b$ for a | 12. $P = 2l + 2w$ for w |
| 13. $F = \frac{9}{5}C + 32$ for C | 14. $2S = n(a + l)$ for a | |

Applying Skills

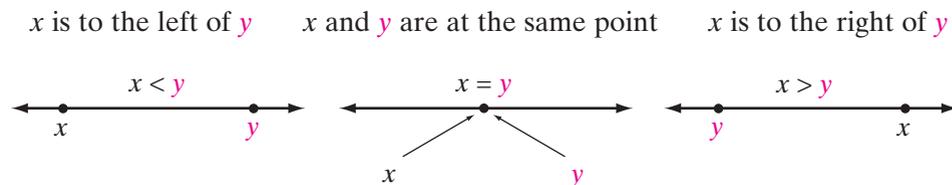
15. The concession stand at a movie theater wants to sell popcorn in containers that are in the shape of a cylinder. The volume of the cylinder is given by the formula $V = \pi r^2 h$, where V is the volume, r is the radius of the base, and h is the height of the container.
- Solve the formula for h .
 - If the container is to hold 1,400 cubic centimeters of popcorn, find, to the *nearest tenth*, the height of the container if the radius of the base is:
 - 4.0 centimeters
 - 5.0 centimeters
 - 8.0 centimeters
 - The concession stand wants to put an ad with a height of 20 centimeters on the side of the container. Which height from part **b** do you think would be the best for the container? Why?

16. A bus travels from Buffalo to Albany, stopping at Rochester and Syracuse. At each city there is a 30-minute stopover to unload and load passengers and baggage. The driving distance from Buffalo to Rochester is 75 miles, from Rochester to Syracuse is 85 miles, and from Syracuse to Albany is 145 miles. The bus travels at an average speed of 50 miles per hour.
- Solve the formula $d = rt$ for t to find the time needed for each part of the trip.
 - Make a schedule for the times of arrival and departure for each city if the bus leaves Buffalo at 9:00 A.M.

4-7 PROPERTIES OF INEQUALITIES

The Order Property of Real Numbers

If two real numbers are graphed on the number line, only one of the following three situations can be true:



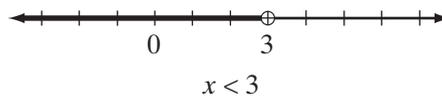
These three cases illustrate the **order property of real numbers**:

- If x and y are two real numbers, then one and only one of the following can be true:

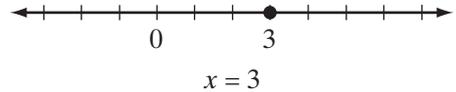
$$x < y \text{ or } x = y \text{ or } x > y$$

Let y be a fixed point, for example, $y = 3$. Then y separates the real numbers into three sets. For any real number, one of the following must be true: $x < 3$, $x = 3$, $x > 3$.

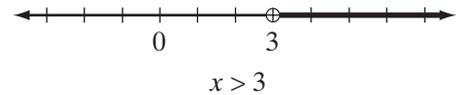
The real numbers, x , that make the inequality $x < 3$ true are to the left of 3 on the number line. The circle at 3 indicates that 3 is the boundary value of the set. The circle is not filled in, indicating that 3 does not belong to this set.



The real number that makes the corresponding equality, $x = 3$, true is a single point on the number line. This point, $x = 3$, is also the boundary between the values of x that make $x < 3$ true and the values of x that make $x > 3$ true. The circle is filled in, indicating that 3 belongs to this set. Here, 3 is the only element of the set.

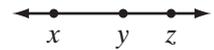


The real numbers, x , that make $x > 3$ true are to the right of 3 on the number line. Again, the circle at 3 indicates that 3 is the boundary value of the set. The circle is not filled in, indicating that 3 does not belong to this set.



The Transitive Property of Inequality

From the graph at the right, you can see that, if x lies to the left of y , and y lies to the left of z , then x lies to the left of z .



The graph illustrates the **transitive property of inequality**:

► For the real numbers x , y , and z :

If $x < y$ and $y < z$, then $x < z$; and if $z > y$ and $y > x$, then $z > x$.

The Addition Property of Inequality

The following table shows the result of adding a number to both sides of an inequality.

True Sentence	Number to Add to Both Sides	Result
$9 > 2$ Order is "greater than."	$9 + 3 ? 2 + 3$ Add a positive number.	$12 > 5$ Order is unchanged.
$9 > 2$ Order is "greater than."	$9 + (-3) ? 2 + (-3)$ Add a negative number.	$6 > -1$ Order is unchanged.
$2 < 9$ Order is "less than."	$2 + 3 ? 9 + 3$ Add a positive number.	$5 < 12$ Order is unchanged.
$2 < 9$ Order is "less than."	$2 + (-3) ? 9 + (-3)$ Add a negative number.	$-1 < 6$ Order is unchanged.

The table illustrates the **addition property of inequality**:

► **For the real numbers x , y , and z :**

If $x < y$, then $x + z < y + z$; and if $x > y$, then $x + z > y + z$.

Since subtracting the same number from both sides of an inequality is equivalent to adding the additive inverse to both sides of the inequality, the following is true:

► **When the same number is added to or subtracted from both sides of an inequality, the order of the new inequality is the same as the order of the original one.**

EXAMPLE 1

Use the inequality $5 < 9$ to write a new inequality:

- a. by adding 6 to both sides
b. by adding -9 to both sides

Solution a. $5 + 6 < 9 + 6$ b. $5 + (-9) < 9 + (-9)$
 $11 < 15$ $-4 < 0$

Answers a. $11 < 15$ b. $-4 < 0$ ■

The Multiplication Property of Inequality

The following table shows the result of multiplying both sides of an inequality by the same number.

True Sentence	Number to Multiply Both Sides	Result
$9 > 2$ Order is "greater than."	$9(3) ? 2(3)$ Multiply by a positive number	$27 > 6$ Order is unchanged.
$5 < 9$ Order is "less than."	$5(3) ? 9(3)$ Multiply by a positive number.	$15 < 27$ Order is unchanged.
$9 > 2$ Order is "greater than."	$9(-3) ? 2(-3)$ Multiply by a negative number	$-27 < -6$ Order is changed.
$5 < 9$ Order is "less than."	$5(-3) ? 9(-3)$ Multiply by a negative number.	$-15 > -27$ Order is changed.

The table illustrates that the order does not change when both sides are multiplied by the same positive number, but does change when both sides are multiplied by the same negative number.

In general terms, the **multiplication property of inequality** states:

► **For the real numbers x , y , and z :**

If z is positive ($z > 0$) and $x < y$, then $xz < yz$.

If z is positive ($z > 0$) and $x > y$, then $xz > yz$.

If z is negative ($z < 0$) and $x < y$, then $xz > yz$.

If z is negative ($z < 0$) and $x > y$, then $xz < yz$.

Dividing both sides of an inequality by a number is equivalent to multiplying both sides by the multiplicative inverse of the number. A number and its multiplicative inverse always have the same sign. Therefore, the following is true:

► **When both sides of an inequality are multiplied or divided by the same *positive* number, the order of the new inequality is the same as the order of the original one.**

► **When both sides of an inequality are multiplied or divided by the same *negative* number, the order of the new inequality is the opposite of the order of the original one.**

EXAMPLE 2

Use the inequality $6 < 9$ to write a new inequality:

a. by multiplying both sides by 2.

b. by multiplying both sides by $-\frac{1}{3}$.

Solution a. $6 < 9$

$$6(2) < 9(2)$$

$$12 < 18$$

b. $6 < 9$

$$6\left(-\frac{1}{3}\right) > 9\left(-\frac{1}{3}\right)$$

$$-2 > -3$$

Answers a. $12 < 18$ b. $-2 > -3$ ■

EXERCISES

Writing About Mathematics

1. Sadie said that if $5 > 4$, then it must be true that $5x > 4x$. Do you agree with Sadie? Explain why or why not.
2. Lucius said that if $x > y$ and $a > b$ then $x + a > y + b$. Do you agree with Lucius? Explain why or why not.
3. Jason said that if $x > y$ and $a > b$ then $x - a > y - b$. Do you agree with Jason? Explain why or why not.

Developing Skills

In 4–31, replace each question mark with the symbol $>$ or $<$ so that the resulting sentence will be true.

4. Since $8 > 2$, $8 + 1$? $2 + 1$.
5. Since $-6 < 2$, $-6 + (-4)$? $2 + (-4)$.
6. Since $9 > 5$, $9 - 2$? $5 - 2$.
7. Since $-2 > -8$, $-2 - \left(\frac{1}{4}\right)$? $-8 - \left(\frac{1}{4}\right)$.
8. Since $7 > 3$, $\frac{2}{3}(7)$? $\frac{2}{3}(3)$.
9. Since $-8 < 4$, $(-8) \div (4)$? $(4) \div (4)$.
10. Since $9 > 6$, $9 \div \left(-\frac{1}{3}\right)$? $6 \div \left(-\frac{1}{3}\right)$.
11. If $5 > x$, then $5 + 7$? $x + 7$.
12. If $y < 6$, then $y - 2$? $6 - 2$.
13. If $20 > r$, then $4(20)$? $4(r)$.
14. If $t < 64$, then $t \div 8$? $64 \div 8$.
15. If $x > 8$, then $-2x$? $-2(8)$.
16. If $y < 8$, then $y \div (-4)$? $8 \div (-4)$.
17. If $x + 2 > 7$, the $x + 2 + (-2)$? $7 + (-2)$ or x ? 5 .
18. If $y - 3 < 12$, then $y - 3 + 3$? $12 + 3$ or y ? 15 .
19. If $a + 5 < 14$, then $a + 5 - 5$? $14 - 5$ or a ? 9 .
20. If $2x > 8$, then $\frac{2x}{2}$? $\frac{8}{2}$ or x ? 4 .
21. If $\frac{1}{3}y < 4$, then $3\left(\frac{1}{3}y\right)$? $3(4)$ or y ? 12 .
22. If $-3x < 36$, then $\frac{-3x}{-3}$? $\frac{36}{-3}$ or x ? -12 .
23. If $-2x > 6$, then $-\frac{1}{2}(-2x)$? $-\frac{1}{2}(6)$ or x ? -3 .
24. If $x < 5$ and $5 < y$, then x ? y .
25. If $m > -7$ and $-7 > a$, then m ? a .
26. If $3 < 7$, then 7 ? 3 .
27. If $-4 > -12$, then -12 ? -4 .
28. If $9 > x$, then x ? 9 .
29. If $-7 < a$, then a ? -7 .
30. If $x < 10$ and $10 < z$, then x ? z .
31. If $a > b$ and $c < b$, then a ? c .

4-8 FINDING AND GRAPHING THE SOLUTION OF AN INEQUALITY

When an inequality contains a variable, the **domain** or **replacement set of the inequality** is the set of all possible numbers that can be used to replace the variable. When an element from the domain is used in place of the variable, the inequality may be true or it may be false. The **solution set of an inequality** is the set of numbers from the domain that make the inequality true. Inequalities that have the same solution set are **equivalent inequalities**.

To find the solution set of an inequality, solve the inequality by methods similar to those used in solving an equation. Use the properties of inequalities to transform the given inequality into a simpler equivalent inequality whose solution set is evident.

In Examples 1–5, the domain is the set of real numbers.

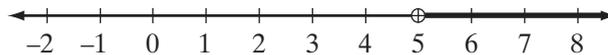
EXAMPLE 1

Find and graph the solution of the inequality $x - 4 > 1$.

Solution

How to Proceed

- | | |
|---|---|
| (1) Write the inequality: | $x - 4 > 1$ |
| (2) Use the addition property of inequality.
Add 4 to each side: | $\begin{array}{r} x - 4 > 1 \\ + 4 \quad + 4 \\ \hline x > 5 \end{array}$ |



The graph above shows the solution set. The circle at 5 indicates that 5 is the boundary between the numbers to the right, which belong to the solution set, and the numbers to the left, which do not belong. Since 5 is not included in the solution set, the circle is not filled in.

- Check**
- (1) Check one value from the solution set, for example, 7.
This value will make the inequality true. $7 - 4 > 1$ is true.
 - (2) Check the boundary value, 5. This value, which separates the values that make the inequality true from the values that make it false, will make the corresponding equality, $x - 4 = 1$, true. $5 - 4 = 1$ is true.

Answer $x > 5$ ■

An alternative method of expressing the solution set is **interval notation**. When this notation is used, the solution set is written as $(5, \infty)$. The first number, 5 names the lower boundary. The symbol ∞ , often called infinity, indicates that there is no upper boundary, that is, that the set of real numbers continues without end. The parentheses indicate that the boundary values are not elements of the set.

EXAMPLE 2

Find and graph the solution of $5x + 4 < 11 - 2x$.

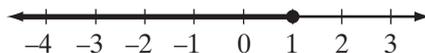
Solution The solution set of $5x + 4 \leq 11 - 2x$ includes all values of the domain for which either $5x + 4 < 11 - 2x$ is true or $5x + 4 = 11 - 2x$ is true.

How to Proceed

(1) Write the inequality:	$5x + 4 \leq 11 - 2x$
(2) Add $2x$ to each side:	$\frac{+2x}{7x + 4} \leq \frac{+2x}{11}$
(3) Add -4 to each side:	$\frac{-4}{7x} \leq \frac{-4}{7}$
(4) Divide each side by 7:	$\frac{7x}{7} \leq \frac{7}{7}$ $x \leq 1$

The solution set includes 1 and all of the real numbers less than 1. This is shown on the graph below by filling in the circle at 1 and drawing a heavy line to the left of 1.

Answer $x \leq 1$



The solution set can also be written in interval notation as $(-\infty, 1]$. The symbol $-\infty$, often called negative infinity, indicates that there is no lower boundary, that is, all negative real numbers less than the upper boundary are included. The number, 1, names the upper boundary. The right bracket indicates that the upper boundary value is an element of the set.

EXAMPLE 3

Find and graph the solution set: $2(2x - 8) - 8x \leq 0$.

Solution

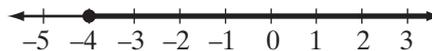
How to Proceed

(1) Write the inequality:	$2(2x - 8) - 8x \leq 0$
(2) Use the distributive property:	$4x - 16 - 8x \leq 0$
(3) Combine like terms in the left side:	$-4x - 16 \leq 0$
(4) Add 16 to each side:	$\frac{+16}{-4x} \leq \frac{+16}{16}$

- (5) Divide both sides by -4 . Dividing by a negative number reverses the inequality:
- (6) The graph of the solution set includes -4 and all of the real numbers to the right of -4 on the number line:

$$\frac{-4x}{-4} \geq \frac{16}{-4}$$

$$x \geq -4$$



Answer $x \geq -4$ or $[-4, \infty)$

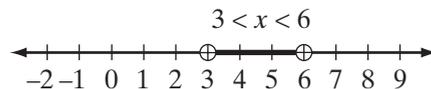
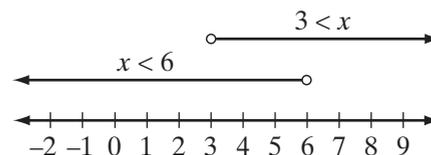
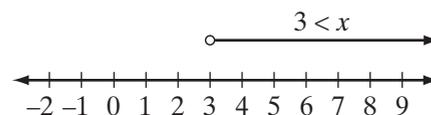
Graphing the Intersection of Two Sets

The inequality $3 < x < 6$ is equivalent to $(3 < x)$ and $(x < 6)$. This statement is true when both simple statements are true and false when one or both statements are false. The solution set of this inequality consists of all of the numbers that are in the solution set of both simple inequalities. The graph of $3 < x < 6$ can be drawn as shown below.

How to Proceed

- Draw the graph of the solution set of the first inequality, $3 < x$, a few spaces above the number line:
- Draw the graph of the solution set of the second inequality, $x < 6$, above the number line, but below the graph of the first inequality:
- Draw the graph of the intersection of these two sets by shading, on the number line, the points that belong to the solution set of both simple inequalities:

Solution



Since 3 is in the solution set of $x < 6$ but not in the solution set of $3 < x$, 3 is not in the intersection of the two sets. Also, since 6 is in the solution set of $3 < x$ but not in the solution set of $x < 6$, 6 is not in the intersection of the two sets. Therefore, the circles at 3 and 6 are not filled in, indicating that these boundary values are not elements of the solution set of $3 < x < 6$.

This set can also be written as $(3, 6)$, a pair of numbers that list the left and right boundaries of the set. The parentheses indicate that the boundary values do not belong to the set. Similarly, the set of numbers $3 \leq x \leq 6$ can be written as $[3, 6]$. The brackets indicate that the boundary values do belong to the set.

Although this notation is similar to that used for an ordered pair that names a point in the coordinate plane, the context in which the interval or ordered pair is used will determine the meaning.

EXAMPLE 4

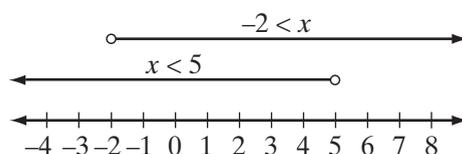
Solve the inequality and graph the solution set: $-7 < x - 5 < 0$.

Solution
How to Proceed

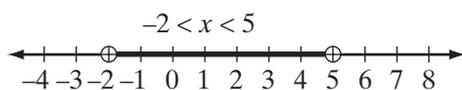
(1) First solve the inequalities for x :

$$\begin{array}{r} -7 < x - 5 < 0 \\ +5 \quad +5 \quad +5 \\ \hline -2 < x < 5 \end{array}$$

(2) Draw the graphs of $-2 < x$ and $x < 5$ above the number line:



(3) Draw the graph of all points that are common to the graphs of $-2 < x$ and $x < 5$:



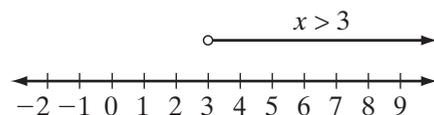
Answer $-2 < x < 5$ or $(-2, 5)$ ■

Graphing the Union of Two Sets

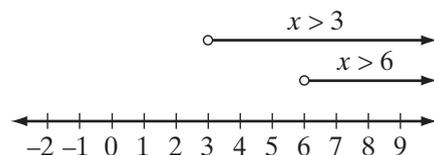
The inequality $(x > 3)$ or $(x > 6)$ is true when one or both of the simple statements are true. It is false when both simple statements are false. The solution set of the inequality consists of the union of the solution sets of the two simple statements. The graph of the solution set can be drawn as shown below.

How to Proceed

(1) Draw the graph of the solution set of the first inequality a few spaces above the number line:



(2) Draw the graph of the solution set of the second inequality above the number line, but below the graph of the first inequality:


Solution

- (3) Draw the graph of the union by shading, on the number line, the points that belong to the solution of one or both of the simple inequalities:



Since 3 is not in the solution set of either inequality, 3 is not in the union of the two sets. Therefore, the circle at 3 is not filled in.

Answer: $x > 3$ or $(3, \infty)$

EXAMPLE 5

Solve the inequality and graph the solution set: $(x + 2 < 0)$ or $(x - 3 > 0)$.

Solution

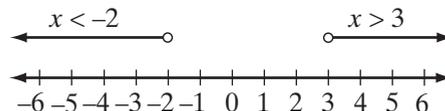
How to Proceed

- (1) Solve each inequality for x :

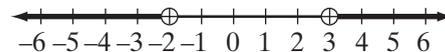
$$x + 2 < 0 \quad \text{or} \quad x - 3 > 0$$

$$\frac{-2}{x} < \frac{-2}{-2} \quad \frac{+3}{x} > \frac{+3}{3}$$

- (2) Draw the graphs of $x < -2$ and $x > 3$ above the number line:



- (3) Draw the graph of all points of the graphs of $x < -2$ or $x > 3$:



Answer $(x < -2)$ or $(x > +3)$ or $(\infty, -2)$ or $(+3, \infty)$ ■

Note: Since the solution is the union of two sets, the answer can also be expressed using set notation: $(x < -2) \cup (x > 3)$ or $(-\infty, -2) \cup (3, \infty)$.

EXERCISES

Writing About Mathematics

- Give an example of a situation that can be modeled by the inequality $x > 5$ in which **a.** the solution set has a smallest value, and **b.** the solution set does not have a smallest value.
- Abram said that the solution set of $(x < 4)$ or $(x > 4)$ is the set of all real numbers. Do you agree with Abram? Explain why or why not.

Developing Skills

In 3–37, find and graph the solution set of each inequality. The domain is the set of real numbers.

- | | | | |
|--------------------------------------|------------------------------------|-------------------------------------|----------------------------|
| 3. $x - 2 > 4$ | 4. $z - 6 < 4$ | 5. $y - \frac{1}{2} > 2$ | 6. $x - 1.5 < 3.5$ |
| 7. $x + 3 > 6$ | 8. $19 < y + 17$ | 9. $d + \frac{1}{4} > 3\frac{1}{4}$ | 10. $-3.5 > c + 0.5$ |
| 11. $y - 4 \geq 4$ | 12. $25 \leq d + 22$ | 13. $3t > 6$ | 14. $2x \leq 12$ |
| 15. $15 \leq 3y$ | 16. $-10 \leq 4h$ | 17. $-6y < 24$ | 18. $27 > -9y$ |
| 19. $-10x > -20$ | 20. $12 \leq -1.2r$ | 21. $\frac{1}{3}x > 2$ | 22. $-\frac{2}{3}z \geq 6$ |
| 23. $\frac{x}{2} > 1$ | 24. $-10 \geq 2.5z$ | 25. $2x - 1 > 5$ | 26. $3y - 6 \geq 12$ |
| 27. $5y + 3 \geq 13$ | 28. $5x - 4 > 4 - 3x$ | 29. $8y - 1 \leq 3y + 29$ | 30. $6x + 2 < 8x + 14$ |
| 31. $8m \geq 2(2m + 3)$ | 32. $0 < x + 3 < 6$ | 33. $-5 \leq x + 2 < 7$ | |
| 34. $0 < 2x - 4 \leq 6$ | 35. $(x - 1 > 3)$ or $(x + 1 > 9)$ | 36. $(2x < 2)$ or $(x + 5 \geq 10)$ | |
| 37. $(x - 5 < 2x)$ or $(x + 8 > 3x)$ | | | |

38. Which of the following is equivalent to $y + 4 \geq 9$?

- (1) $y > 5$ (2) $y \geq 5$ (3) $y \geq 13$ (4) $y = 13$

39. Which of the following is equivalent to $4x < 5x + 6$?

- (1) $x > -6$ (2) $x < -6$ (3) $x > 6$ (4) $x < 6$

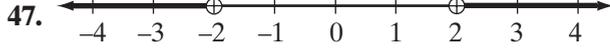
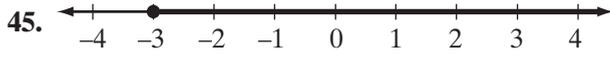
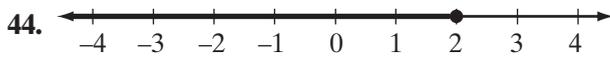
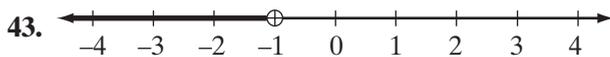
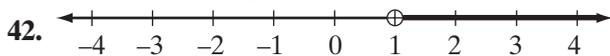
40. The smallest member of the solution set of $3x - 7 \geq 8$ is

- (1) 3 (2) 4 (3) 5 (4) 6

41. The largest member of the solution set of $4x \leq 3x + 2$ is

- (1) 1 (2) 2 (3) 3 (4) 4

In 42–47, write an inequality for each graph using interval notation.



48. a. Graph the inequality $(x > 2)$ or $(x = 2)$.

b. Write an inequality equivalent to $(x > 2)$ or $(x = 2)$.

4-9 USING INEQUALITIES TO SOLVE PROBLEMS

Many problems can be solved by writing an inequality that describes how the numbers in the problem are related and then solving the inequality. An inequality can be expressed in words in different ways. For example:

$$x > 12$$

A number is more than 12.
 A number exceeds 12.
 A number is greater than 12.
 A number is over 12.

$$x \geq 12$$

A number is at least 12.
 A number has a minimum value of 12.
 A number is not less than 12.
 A number is not under 12.

$$x < 12$$

A number is less than 12.
 A number is under 12.

$$x \leq 12$$

A number is at most 12.
 A number has a maximum value of 12.
 A number is not greater than 12.
 A number does not exceed 12.
 A number is not more than 12.

Procedure

To solve a problem that involves an inequality:

1. Choose a variable to represent one of the unknown quantities in the problem.
2. Express other unknown quantities in terms of the same variable.
3. Choose an appropriate domain for the problem.
4. Write an inequality using a relationship given in the problem, a previously known relationship, or a formula.
5. Solve the inequality.
6. Check the solution using the words of the problem.
7. Use the solution of the inequality to answer the question in the problem.

EXAMPLE 1

Serafina has \$53.50 in her pocket and wants to purchase shirts at a sale price of \$14.95 each. How many shirts can she buy?

Solution (1) Choose a variable to represent the number of shirts Serafina can buy and the cost of the shirts.

Let x = the number of shirts that she can buy.

Then, $14.95x$ = the cost of the x shirts.

The domain is the set of whole numbers, since she can only buy a whole number of shirts.

- (2) Write an inequality using a relationship given in the problem.

The cost of the shirts is less than or equal to \$53.50.

$$14.95x \leq 53.50$$

- (3) Solve the inequality.

$$14.95x \leq 53.50$$

$$\frac{14.95x}{14.95} \leq \frac{53.50}{14.95}$$

Use a calculator to complete the computation.

ENTER: 53.50 \div 14.95 **ENTER**

DISPLAY: $\boxed{53.50/14.95}$
 $\boxed{3.578595318}$

Therefore, $x \leq 3.578595318$. Since the domain is the set of whole numbers, the solution set is $\{x : x \text{ is a counting number less than or equal to } 3\}$ or $\{0, 1, 2, 3\}$.

- (4) Check the solution in the words of the problem.

0 shirt costs $\$14.95(0) = \0

1 shirt costs $\$14.95(1) = \14.95

2 shirts cost $\$14.95(2) = \29.90

3 shirts cost $\$14.95(3) = \44.85

4 or more shirts cost more than $\$14.95 \cdot (4)$ or more than \$59.80

Answer Serafina can buy 0, 1, 2, or 3 shirts. ■

EXAMPLE 2

The length of a rectangle is 5 centimeters more than its width. The perimeter of the rectangle is at least 66 centimeters. Find the minimum measures of the length and width.

Solution If the perimeter is at least 66 centimeters, then the sum of the measures of the four sides is either equal to 66 centimeters or is greater than 66 centimeters.

Let x = the width of the rectangle.

Then, $x + 5$ = the length of the rectangle.

The domain is the set of positive real numbers.

The perimeter of the rectangle is at least 66 centimeters.

$$x + (x + 5) + x + (x + 5) \geq 66$$

$$4x + 10 \geq 66$$

$$4x + 10 - 10 \geq 66 - 10$$

$$4x \geq 56$$

$$x \geq 14$$

The width can be any real number that is greater than or equal to 14 and the length is any real number that is 5 more than the width. Since we are looking for the minimum measures, the smallest possible width is 14 and the smallest possible length is $14 + 5$ or 19.

Answer The minimum width is 14 centimeters, and the minimum length is 19 centimeters. ▣

EXERCISES

Writing About Mathematics

- If there is a number x such that $-x < -3$, is it true that $x > -3$? Explain why or why not.
- Is the solution set of $-x < -3$ the same as the solution set of $x > -3$? Explain why or why not.

Developing Skills

In 3–12, represent each sentence as an algebraic inequality.

- | | |
|--|--|
| 3. x is less than or equal to 15. | 4. y is greater than or equal to 4. |
| 5. x is at most 50. | 6. x is more than 50. |
| 7. The greatest possible value of $3y$ is 30. | 8. The sum of $5x$ and $2x$ is at least 70. |
| 9. The maximum value of $4x - 6$ is 54. | 10. The minimum value of $2x + 1$ is 13. |
| 11. The product of $3x$ and $x + 1$ is less than 35. | 12. When x is divided by 3 the quotient is greater than 7. |

In 13–19, in each case write and solve the inequality that represents the given conditions. Use n as the variable.

- | | |
|--|--|
| 13. Six less than a number is less than 4. | 14. Six less than a number is greater than 4. |
| 15. Six times a number is less than 72. | 16. A number increased by 10 is greater than 50. |

17. A number decreased by 15 is less than 35.
18. Twice a number, increased by 6, is less than 48.
19. Five times a number, decreased by 24, is greater than 3 times the number.

Applying Skills

In 20–29, in each case write an inequality and solve the problem algebraically.

20. Mr. Burke had a sum of money in a bank. After he deposited an additional sum of \$100, he had at least \$550 in the bank. At least how much money did Mr. Burke have in the bank originally?
21. The members of a club agree to buy at least 250 tickets for a theater party. If they expect to buy 80 fewer orchestra tickets than balcony tickets, what is the least number of balcony tickets they will buy?
22. Mrs. Scott decided that she would spend no more than \$120 to buy a jacket and a skirt. If the price of the jacket was \$20 more than 3 times the price of the skirt, find the highest possible price of the skirt.
23. Three times a number increased by 8 is at most 40 more than the number. Find the greatest value of the number.
24. The length of a rectangle is 8 meters less than 5 times its width. If the perimeter of the rectangle is at most 104 meters, find the greatest possible width of the rectangle.
25. The length of a rectangle is 12 centimeters less than 3 times its width. If the perimeter of the rectangle is at most 176 centimeters, find the greatest possible length of the rectangle.
26. Mrs. Diaz wishes to save at least \$1,500 in 12 months. If she saves \$300 during the first 4 months, what is the least possible average amount that she must save in each of the remaining 8 months?
27. Two consecutive even numbers are such that their sum is greater than 98 decreased by twice the larger. Find the smallest possible values for the integers.
28. Minou wants \$29 to buy music online. Her father agrees to pay her \$6 an hour for gardening in addition to her \$5 weekly allowance for helping around the house. What is the minimum number of hours Minou must work at gardening to receive at least \$29 this week?
29. Allison has more than 2 but less than 3 hours to spend on her homework. She has work in math, English, and social studies. She plans to spend equal amounts of time studying English and studying social studies, and to spend twice as much time studying math as in studying English.
- What is the minimum number of minutes she can spend on English homework?
 - What is the maximum number of minutes she can spend on social studies?
 - What is the maximum number of minutes she can devote to math?

CHAPTER SUMMARY

The properties of equality allow us to write equivalent equations to solve an equation.

- 1. The addition property of equality:** If equals are added to equals the sums are equal.
- 2. The subtraction property of equality:** If equals are subtracted from equals the differences are equal.
- 3. The multiplication property of equality:** If equals are multiplied by equals the products are equal.
- 4. The division property of equality:** If equals are divided by nonzero equals the quotients are equal.
- 5. The substitution property:** In any statement of equality, a quantity may be substituted for its equal.

Before solving an equation, simplify each side if necessary. To solve an equation that has the variable in both sides, transform it into an equivalent equation in which the variable appears in only one side. Do this by adding the opposite of the variable term on one side to both sides of the equation. Use the properties of equality.

Any equation or formula containing two or more variables can be transformed so that one variable is expressed in terms of all other variables. To do this, think of solving a simpler but related equation that contains only one variable.

Order property of numbers: For real numbers x and y , one and only one of the following can be true: $x < y$, $x = y$, or $x > y$.

Transitive property of inequality: For real numbers x , y , and z , if $x < y$ and $y < z$, then $x < z$, and if $x > y$ and $y > z$, then $x > z$.

Addition property of inequality: When the same number is added to or subtracted from both sides of an inequality, the order of the new inequality is the same as the order of the original one.

Multiplication property of inequality: When both sides of an inequality are multiplied or divided by the same positive number, the order of the new inequality is the same as the order of the original one. When both sides of an inequality are multiplied or divided by the same negative number, the order of the new inequality is the opposite of the order of the original one.

The **domain** or **replacement set** of an inequality is the set of all possible numbers that can be used to replace the variable. The **solution set** of an inequality is the set of numbers from the domain that make the inequality true. Inequalities that have the same solution set are **equivalent inequalities**.

VOCABULARY

- 4-1 Equation • Left side • Left member • Right side • Right member • Root • Solution • Solution set • Identity • Equivalent equations • Solve an equation • Conditional equation • Check
- 4-2 First degree equation in one variable • Like terms • Similar terms • Unlike terms
- 4-4 Perimeter • Distance formula
- 4-7 Order property of the real numbers • Transitive property of inequality • Addition property of inequality • Multiplication property of inequality
- 4-8 Domain of an inequality • Replacement set of an inequality • Solution set of an inequality • Equivalent inequalities • Interval notation

REVIEW EXERCISES

1. Compare the properties of equality with the properties of inequality. Explain how they are alike and how they are different.

In 2–9, solve for the variable and check.

- | | |
|-------------------------------|---------------------------------|
| 2. $8w = 60 - 4w$ | 3. $8w - 4w = 60$ |
| 4. $4h + 3 = 23 - h$ | 5. $5y + 3 = 2y$ |
| 6. $8a - (6a - 5) = 1$ | 7. $2(b - 4) = 4(2b + 1)$ |
| 8. $3(4x - 1) - 2 = 17x + 10$ | 9. $(x + 2) - (3x - 2) = x + 3$ |

In 10–15, solve each equation for x in terms of a , b , and c .

- | | | |
|--------------------------|------------------------|-----------------------|
| 10. $a + x = bc$ | 11. $cx + a = b$ | 12. $bx - a = 5a + c$ |
| 13. $\frac{a+c}{2}x = b$ | 14. $\frac{ax}{b} = c$ | 15. $ax + 2b = c$ |

16. a. Solve $A = \frac{1}{2}bh$ for h in terms of A and b .

b. Find h when $A = 5.4$ and $b = 0.9$.

17. If $P = 2l + 2w$, find w when $P = 17$ and $l = 5$.

18. If $F = \frac{9}{5}C + 32$, find C when $F = 68$.

In 19–26, find and graph the solution set of each inequality.

- | | | |
|-----------------|-------------------------|--------------------------------------|
| 19. $6 + x > 3$ | 20. $2x - 3 \geq -5$ | 21. $\frac{1}{3}x < 1$ |
| 22. $-x \geq 4$ | 23. $-3 < x - 1 \leq 2$ | 24. $(x + 2 \geq 5)$ and $(2x < 14)$ |

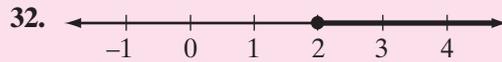
25. $(-x \geq 2)$ or $(x > 0)$ 26. $(x - 4 \geq 1)$ and $(-2x > -18)$

In 27–30, tell whether each statement is sometimes, always, or never true. Justify your answer by stating a property of inequality or by giving a counterexample.

27. If $x > y$, then $a + x > a + y$. 28. If $x > y$, then $ax > ay$.
 29. If $x > y$ and $y > z$, then $x > z$. 30. If $x > y$, then $-x > -y$.

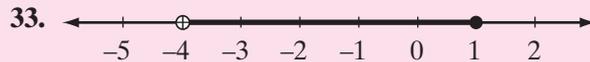
In 31–33, select the answer choice that correctly completes the statement or answers the question.

31. An inequality that is equivalent to $4x - 3 > 5$ is
 (1) $x > 2$ (2) $x < 2$ (3) $x > \frac{1}{2}$ (4) $x < \frac{1}{2}$



The solution set of which inequality is shown in the graph above?

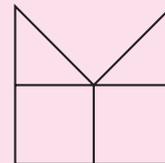
- (1) $x - 2 \geq 0$ (2) $x - 2 > 0$ (3) $x - 2 < 0$ (4) $x - 2 \leq 0$



The above graph shows the solution set of which inequality?

- (1) $-4 < x < 1$ (2) $-4 \leq x < 1$ (3) $-4 < x \leq 1$ (4) $-4 \leq x \leq 1$

34. The figure on the right consists of two squares and two isosceles right triangles. Express the area of the figure in terms of s , the length of one side of a square.

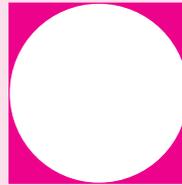


35. Express in terms of w the number of days in w weeks and 4 days.
 36. The length of a rectangular room is 5 feet more than 3 times the width. The perimeter of the room is 62 feet. Find the dimensions of the room.
 37. A truck must cross a bridge that can support a maximum weight of 24,000 pounds. The weight of the empty truck is 1,500 pounds, and the driver weighs 190 pounds. What is the weight of a load that the truck can carry?
 38. In an apartment building there is one elevator, and the maximum load that it can carry is 2,000 pounds. The maintenance supervisor wants to move a replacement part for the air-conditioning unit to the roof. The part weighs 1,600 pounds, and the mechanized cart on which it is being moved weighs 250 pounds. When the maintenance supervisor drives the cart onto the elevator, the alarm sounds to signify that the elevator is overloaded.

- a. How much does the maintenance supervisor weigh?
 b. How can the replacement part be delivered to the roof if the part cannot be disassembled?
39. A mail-order digital photo developer charges 8 cents for each print plus a \$2.98 shipping fee. A local developer charges 15 cents for each print. How many digital prints must be ordered in order that:
- a. the local developer offers the lower price?
 b. the mail-order developer offers the lower price?
40. a. What is an appropriate replacement set for the problem in the chapter opener on page 116 of this chapter?
 b. Write and solve the equations suggested by this problem.
 c. Write the solution set for this problem.

Exploration

The figure at the right shows a circle inscribed in a square. Explain how this figure shows that $\pi r^2 < (2r)^2$.



CUMULATIVE REVIEW

CHAPTERS 1–4

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. The rational numbers are a subset of
 (1) the integers (3) the whole numbers
 (2) the counting numbers (4) the real numbers
2. If $x - 12.6 = 8.4 + 0.7x$, then x equals
 (1) 0.07 (2) 0.7 (3) 7 (4) 70
3. The solution set of $2x - 4 = 5x + 14$ is
 (1) $\left\{-\frac{10}{3}\right\}$ (2) $\left\{\frac{10}{3}\right\}$ (3) $\{6\}$ (4) $\{-6\}$
4. Which of the following inequalities is false?
 (1) $\frac{2}{3} \leq 0.6$ (2) $\frac{2}{3} \leq 0.\bar{6}$ (3) $\frac{2}{3} \neq 0.6$ (4) $\frac{2}{3} \geq 0.6$

5. Which of the following identities is an illustration of the commutative property of addition?
(1) $(x + 3) + 2 = x + (3 + 2)$ (3) $5(x + 3) = 5x + 15$
(2) $x + 3 = 3 + x$ (4) $x + 0 = x$
6. Which of the following sets is closed under division?
(1) nonzero whole numbers (3) nonzero even integers
(2) nonzero integers (4) nonzero rational numbers
7. The measure of one side of a rectangle is 20.50 feet. This measure is given to how many significant digits?
(1) 1 (2) 2 (3) 3 (4) 4
8. In the coordinate plane, the vertices of quadrilateral $ABCD$ have the coordinates $A(-2, 0)$, $B(7, 0)$, $C(7, 5)$, and $D(0, 5)$. The quadrilateral is
(1) a rhombus (3) a parallelogram
(2) a rectangle (4) a trapezoid
9. One element of the solution set of $(x \leq -3)$ or $(x > 5)$ is
(1) -4 (2) -2 (3) 5
(4) None of the above. The solution set is the empty set.
10. When $x = -3$, $-x^2$ is
(1) -6 (2) 6 (3) -9 (4) 9

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

11. A quadrilateral has four sides. Quadrilateral $ABCD$ has three sides that have equal measures. The measure of the fourth side is 8.0 cm longer than each of the other sides. If the perimeter of the quadrilateral is 28.0 m, find the measure of each side using the correct number of significant digits.
12. To change degrees Fahrenheit, F , to degrees Celsius, C , subtract 32 from the Fahrenheit temperature and multiply the difference by five-ninths.
- Write an equation for C in terms of F .
 - Normal body temperature is 98.6° Fahrenheit. What is normal body temperature in degrees Celsius?

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

13. Is it possible for the remainder to be 2 when a prime number that is greater than 2 is divided by 4? Explain why or why not.
14. A plum and a pineapple cost the same as three peaches. Two plums cost the same as a peach. How many plums cost the same as a pineapple?

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

15. A trapezoid is a quadrilateral with only one pair of parallel sides called the bases of the trapezoid. The formula for the area of a trapezoid is $A = \frac{h}{2}(b_1 + b_2)$ where h represents the measure of the altitude to the bases, and b_1 and b_2 represent the measures of the bases. Find the area of a trapezoid if $h = 5.25$ cm, $b_1 = 12.75$ cm, and $b_2 = 9.50$ cm. Express your answer to the number of significant digits determined by the given data.
16. Fred bought three shirts, each at the same price, and received less than \$12.00 in change from a \$50.00 bill.
 - a. What is the minimum cost of one shirt?
 - b. What is the maximum cost of one shirt?